Capital accumulation under different financial agreements

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This paper develops a simple overlapping-generations model where agents' income is given both by a stochastic endowment and by the profits generated by a production activity. The purpose is to analyze the consequences of different forms of financial agreements on capital accumulation. In this model, Pareto optimality requires that the capital stock is a deterministic function of the previous level of capital. Agents can eliminate any randomness in the capital-accumulation process when contingent claims markets are available. When standard loan contracts prevail because of asymmetric information, the economy incurs an efficiency loss due to capital-stock fluctuations. The expected level of capital under this last regime is always smaller than the one achieved when markets for contingent claims exist.

1. Introduction

It is well known that under uncertainty, financial markets not only have the role of transferring funds from the owners of financial wealth to the entrepreneurs, but also have the role of allocating risk among them. Financial agreements, working as risk-sharing devices, may affect the dynamics of an economy and have important effects on capital accumulation. In this paper, we will consider the case of an economy where investment and capital accumulation show relevant fluctuations due to the stochastic environment, and we will try to understand how these fluctuations are related to the structure of prevailing financial contracts. In particular, we will try to compare a situation in which financial markets are incomplete and standard loan contracts prevail with a situation in which financial markets are complete.

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In recent years, some important results in economic theory have been related to the absence of complete markets for contingent claims. In monetary overlapping-generations models, for example, market incompleteness allows asset prices to depart from market fundamentals, so that some equilibria are characterized by rational bubbles [Tirole (1985)]. In the stochastic model proposed by Cass and Shell (1983) sunspots affect consumption levels only if some individuals are excluded from trading contingent securities based on sunspots. Hardly any attention, however, has ever been given to the effects of market incompleteness on a dynamic economy with production. This paper suggests that some of the aggregate instability observed in the process of capital accumulation can be related to the existence of financial regimes other than the market for contingent claims.

The study will be performed in the context of an overlapping-generations model, where agents are risk-neutral and there is only aggregate risk in the economy. Agents live for three periods. In the first period of their lives, their income is given both by a random endowment and the profits generated by a production activity. In the second period, agents invest their income by lending in the market. In the third period, agents consume all the proceeds from the investments made in the previous period.

In this model any Pareto-optimal allocation is such that the stock of capital in every period is a deterministic function of the previous level of capital. If a market for contingent claims exists, agents, who are assumed to be risk-neutral, will prevent the capital stock from being affected by the stochastic shocks that characterize the economy. In this case, the elimination of any randomness in the process of capital accumulation is due to intertemporal efficiency in production and the fact that equilibrium in the asset market requires a constant, uniform rate of return on each asset.

We then analyze a situation in which, because of asymmetric information and monitoring costs, standard loans are an optimal contractual arrangement. In this event the borrowers, who are also the investors in this economy, are unable to insure their income against random fluctuations and therefore the supply of capital shows persistent fluctuations. Since Pareto optimality requires that the process of capital accumulation be deterministic, these fluctuations represent an inefficiency that standard loan contracts impose on the economy.

We then compare the processes of capital accumulation that arise under the two types of financial regimes and we show that the level of capital that prevails when contingent-claims markets exist is always greater than the expected capital stock that prevails under standard loan contracts.

1This assumption simplifies very much the structure of the financial markets that we consider, since in the model there is no risk that can be diversified. On the other hand, the study will give some insights on the different risk-sharing agreements that can be used to deal with the problem of aggregate risk.
In section 2 we set up the model and we show the properties of any Pareto-optimal allocation; in section 3 we study the model when agents have access to markets for contingent claims; in section 4 we study the economy when standard loan contracts prevail; and in section 5 we compare the dynamic processes derived in the previous sections.

2. The model

We study a simple overlapping-generations model consisting of a single good, where agents are identical and live for three periods. In the first period of their lives, agents have a production technology with which the good is transformed from period $t$ to period $t + 1$. At the beginning of period $t + 1$, agents also receive an endowment $e_{t+1}$ which is a random variable identically and independently distributed across periods according to a probability distribution function $F(e_r)$ with support $[0, e^*]$. Let $\bar{e}$ denote the mean of $e_r$.

Agents are risk-neutral, expected value maximizers, and do not consume either in the first period or in the second period of their lives but only in the third period. We denote by $c_i(h_i)$ the amount consumed by an agent in period $t$ given the history of the economy up to period $t$, $h_t = (e_t, e_{t-1}, e_{t-2}, \ldots)$, and by $k_i(h_i)$ the capital stock chosen by the young agent in period $t$, given $h_t$. Output in period $t$ is given by a production function $g_i(k_{t-1}(h_{t-1})),^2$ such that $g_i^i(k_{t-1}(h_{t-1})) > 0$, $g_i^i(k_{t-1}(h_{t-1})) < 0$. In order to simplify the analysis we also assume

\[ g_i^i(k_{t-1}(h_{t-1})) \geq \bar{e}/k_{t-1}(h_{t-1}) \quad \text{for every} \quad k_{t-1}(h_{t-1}). \quad (1) \]

We immediately establish:

**Proposition 1.** If \{$(c_i(h_i), k_i(h_i))_{i=0}^{\infty}$\} is a Pareto-optimal allocation, then $k_i(h_{t-1}, h_i) = E(k_i(h_i)|k_{t-1})$.

**Proof.** Assume an allocation \{$(c_i(h_i), k_i(h_i))_{i=0}^{\infty}$\} is Pareto-optimal but $k_j(h_j) - E(k_j(h_j)|k_j) \neq 0$ for some $j > 0$. Feasibility requires

\[ c_{t+1}(h_{t+1}) = e_{t+1} + g_{t+1}(k_t(h_t)) - k_{t+1}(h_{t+1}) \quad \text{for every} \quad t. \quad (2) \]

Define $\bar{k}_j = E(k_j(h_j)|k_{t-1}(h_{t-1}))$ and define an allocation \{$(c_i'(h_i), k_i'(h_i))_{i=0}^{\infty}$\} such that $c_i'(h_i) = c_i(h_i)$ for every $t \neq j, j + 1$, $c_j'(h_j) = c_j(h_j) + k_j(h_j) - \bar{k}_j$, $c_{j+1}'(h_{j+1}) = e_{j+1} + g_{j+1}(k_{j}) - k_{j+1}(h_{j+1})$, $k_i'(h_i) = k_i(h_i)$ for every $t \neq j$, and

---

2 The results we obtain in this paper could be derived also by assuming that agents have no endowment of the good and that uncertainty is represented by a multiplicative shock to production. The assumptions made in this paper, however, allow us to convey the same type of result and to avoid some mathematical complexities in the analysis.
$k'_j(h_j) = \bar{k}_j$. Notice that $\{c'_j(h_j), k'_j(h_j)\}_{j=0}^{\infty}$ is a feasible allocation and that $E(c'_j(h_j)) = E(c_j(h_j))$. Given (1.2), since $g_{j,t+1}(k_j(h_j))$ is a strictly concave function,

$$E(c_{j+1}(h_{j+1})) - E(c_j(h_j)) = \left[ g_{j+1}(\bar{k}_j) - Eg_{j+1}(k_j(h_j)) \right] > 0. \quad (3)$$

Since $E(c'_j(h_j)) = E(c_j(h_j))$ for every $t \neq j$, it follows that $\{c_j(h_j), k_j(h_j)\}_{j=0}^{\infty}$ cannot be a Pareto-optimal allocation, which implies a contradiction. 

The result obtained in Proposition 1 is a consequence of the fact that, given the concavity of the production function, the level of output produced with a deterministic level of capital is always greater than the expected level of output produced when capital is random. It follows that agents can always increase their expected consumption if there exist, in the market, institutional arrangements that allow them to eliminate fluctuations in the stock of capital.

Given the results of Proposition 1, which we will use as a benchmark in our analysis, we will now study our model under two different regimes. In the first regime complete markets are available; in the second regime, because of asymmetric information and monitoring costs, financial markets are incomplete and only standard loan contracts are possible between owners of financial wealth and entrepreneurs.

3. Complete markets

In this section we analyze a model where agents, in the first period of their life, finance their production by selling claims on their future income conditional on the realization of $\varepsilon_{t+1}$. Since we are assuming a continuum of states, there will be a continuum of markets for each possible state of the world and a continuum of prices for each possible realization of the output of the firm. In the second period of his life, the representative agent, with the income obtained in the previous period, acquires claims on future production from the currently young agents. In the third period of his life, the agent consumes the proceeds from the previous investment.

We denote by $a_j^t(\varepsilon_{t+1}; h_t)$ a promise made by a young agent in period $t$ to deliver a unit of output in period $t + 1$ conditional on the realization of $\varepsilon_{t+1}$, given the history of the economy up to period $t$. We call $a_j^t(\varepsilon_{t+1}; h_t)$ the claim on future output conditional upon $\varepsilon_{t+1}$ demanded by the middle-aged agent, given $h_t$. We also denote by $p_j^t(\varepsilon_{t+1}; h_t)$ the price of the claim conditional on the realization of $\varepsilon_{t+1}$, given $h_t$, in terms of the consumption good in period $t$. 
Let us consider the relations that must hold in our simple economy in period $t$. Since young agents have no endowment and do not consume, they must issue claims on future output such that

$$k_t(h_t) = \int_0^{\varepsilon_t} a^d_t(\varepsilon_{t+1}; h_t) p_t(\varepsilon_{t+1}; h_t) d\varepsilon_{t+1}. \quad (4)$$

Upon reaching middle age, the assets of an agent when state $\varepsilon_t$ is realized are

$$\pi_t(h_t) = \varepsilon_t + g_t(k_{t-1}) - a^s_t(\varepsilon_t; h_{t-1}). \quad (5)$$

Since middle-aged agents do not consume, but invest all their assets in claims on future output,

$$\pi_t(h_t) = \int_0^{\varepsilon_t} a^d_t(\varepsilon_{t+1}; h_t) p_t(\varepsilon_{t+1}; h_t) d\varepsilon_{t+1}. \quad (6)$$

Old agents consume all the proceeds of their previous investment, so that their consumption in period $t$, given the realizations of $\varepsilon_t$ and $\varepsilon_{t-1}$, is

$$c_t(\varepsilon_t; h_{t-1}) = a^d_{t-1}(\varepsilon_t; h_{t-1}). \quad (7)$$

For every possible realization of $\varepsilon_{t+1}$ the asset market must clear, that is

$$a^d_t(\varepsilon_{t+1}; h_t) = a^s_t(\varepsilon_{t+1}; h_t) \quad \text{for every } \varepsilon_{t-1}. \quad (8)$$

Given eqs. (5), (6), and (7), we can now write the problem of the young agent in period $t$ as that of choosing the level of $k_t(h_t)$, $a^s_t(\cdot; h_t)$, and $a^d_{t+1}(\cdot; \varepsilon_{t+1}, h_t)$ which maximizes, given $p_t(\cdot; h_t)$ and $p_{t+1}(\cdot; \varepsilon_{t+1}, h_t)$,

$$\int_0^{\varepsilon_t} \left[ \int_0^{\varepsilon_t} a^d_{t+1}(\varepsilon_{t+2}, \varepsilon_{t+1}; h_t) f(\varepsilon_{t+2}) d\varepsilon_{t+2} \right] f(\varepsilon_{t+1}) d\varepsilon_{t+1}, \quad (9)$$

subject to

$$\int_0^{\varepsilon_t} a^s_t(\varepsilon_{t+1}; h_t) p_t(\varepsilon_{t+1}; h_t) d\varepsilon_{t+1} = k_t(h_t), \quad (10)$$

$$\int_0^{\varepsilon_t} a^d_{t+1}(\varepsilon_{t+2}, \varepsilon_{t+1}; h_t) p_{t+1}(\varepsilon_{t+2}, \varepsilon_{t+1}; h_t) d\varepsilon_{t+2}$$

$$= \varepsilon_{t+1} + g_{t+1}(k_t) - a^s_t(\varepsilon_{t+1}; h_t), \quad (11)$$

$$a^d_{t+1}(\varepsilon_{t+2}, \varepsilon_{t+1}; h_t) \geq 0. \quad (12)$$
The intertemporal optimization problem of the agent can be solved recursively by breaking it down into two distinct problems: the second-period problem of choosing how many claims to buy, given the previous investment decision and the realization of the shock, and the first-period problem of choosing the capital stock and the amount of claims that must be sold in order to finance the production activity. In order to solve (9)-(12) we first analyze the agent's second-period problem, i.e., we assume that the agent, after having observed the realization of the shock \( \epsilon_{t+1} \) and the profits earned with the previous productive activity, \( \pi_{t}(h_{t+1}) \), chooses the level of \( a_{t+1}^{d}(\epsilon_{t+2}; h_{t+1}) \) to maximize

\[
\int_{0}^{\infty} a_{t+1}^{d}(\epsilon_{t+2}; h_{t+1}) f(\epsilon_{t+2}) \, d\epsilon_{t+2},
\]

subject to

\[
\int_{0}^{\infty} a_{t+1}^{d}(\epsilon_{t+2}; h_{t+1}) p_{t+1}(\epsilon_{t+2}; h_{t+1}) \, d\epsilon_{t+2} = \pi_{t+1}(h_{t+1}).
\]

Let \( v_{t+1}(\pi_{t+1}; h_{t+1}) \) denote the maximum value achieved by eq. (13), given constraint (14). We can establish:

**Lemma 1.** (i) The ratio \( f(\epsilon_{t+2})/p_{t+1}(\epsilon_{t+2}; h_{t+1}) \) is constant and independent of \( \epsilon_{t+2} \). (ii) \( v_{t+1}(\pi_{t+1}; h_{t+1}) = \rho_{t+1}(h_{t+1}) \pi_{t+1} \).

**Proof.** (i) From the first-order conditions of problem (13)-(14) we obtain

\[
f(\epsilon_{t+2})/p_{t+1}(\epsilon_{t+2}; h_{t+1}) = \rho_{t+1}(h_{t+1}) \text{ for every } \epsilon_{t+2},
\]

where \( \rho_{t+1}(h_{t+1}) \) is constant for every \( \epsilon_{t+2} \).

(ii) Substituting (15) into (13) we obtain

\[
\int_{0}^{\infty} a_{t+1}^{d}(\epsilon_{t+2}; h_{t+1}) f(\epsilon_{t+2}) \, d\epsilon_{t+2} = \pi_{t+1}(h_{t+1}) \rho_{t+1}(h_{t+1})
\]

\[
= v_{t+1}(\pi_{t+1}; h_{t+1}),
\]

Eq. (15) tells us that the price of a claim on future output, contingent on the realization of the random variable \( \epsilon_{t+2} \), is proportional to the probability that such a realization will occur. We can interpret \( \rho_{t+1}(h_{t+1}) \) as a rate of return that an agent obtains from acquiring a claim on future output. Eq. (15) implies, therefore, that in equilibrium, prices adjust in such a way that the
rate of return is the same for all such assets. Given this crucial result we can now prove:

**Proposition 2.** \( k_{t+1}(e_{t+1}; h_t) = \bar{e} + g_{t+1}(k_t(h_t)) - g'_{t+1}(k_t(h_t))k_t(h_t). \)

**Proof.** Let us first consider problem (9)-(11), disregarding constraint (12). Since, by Lemma 1, \( v_{t+1}(\pi_{t+1}; h_{t+1}) = \pi_{t+1}(h_{t+1})\rho_{t+1}(h_{t+1}), \) and given eq. (5), problem (9)-(11) can be rewritten as the problem of choosing the level of \( k_t(h_t) \) and the level of \( \alpha_t(e_{t+1}; h_t) \) to maximize

\[
\int_0^{e^*} \rho_{t+1}(e_{t+1}; h_t) \left[ \epsilon_{t+1} + g_{t+1}(k_t(h_t)) - \alpha_t(e_{t+1}; h_t) \right] f(e_{t+1}) \, de_{t+1},
\]

subject to

\[
\int_0^{e^*} \alpha_t^2(e_{t+1}; h_t) \rho_t(e_{t+1}; h_t) \, de_{t+1} = k_t(h_t).
\]

From the first-order conditions, we obtain

\[
f(e_{t+1}) \rho_{t+1}(e_{t+1}, h_t) - \lambda(h_t) \rho_t(e_{t+1}; h_t) = 0,
\]

\[
g'_{t+1}(k_t(h_t)) \int_0^{e^*} \rho_{t+1}(e_{t+1}; h_t) f(e_{t+1}) \, de_{t+1} = \lambda(h_t).
\]

Substituting (19) in (20), we get

\[
g'_{t}(k_t(h_t)) \int_0^{e^*} \rho_t(e_{t+1}; h_t) \, de_{t+1} = 1.
\]

Given (15), also in period \( t \), equilibrium implies that

\[
f(e_{t+1}) - \rho_t(h_t) \rho_t(e_{t+1}; h_t) = 0.
\]

Substituting (22) in (21), we obtain

\[
g'_{t+1}(k_t(h_t)) = \rho_t(h_t).
\]

Also in period \( t + 1 \), we see that

\[
g'_{t+2}(k_{t+1}(h_{t+1})) = \rho_{t+1}(e_{t+1}; h_t).
\]
We now notice that, given (22) and (19) can be rewritten as
\[ \rho_{t+1}(e_{t+1}; h_t) = \lambda(h_t) / \rho_t(h_t) \quad \text{for every } e_{t+1}. \] (25)

From (25) we know that \( \rho_{t+1}(e_{t+1}; h_t) \) is a deterministic function of \( \rho_t(h_t) \) and \( \lambda(h_t) \). Given (23) and (24), \( \rho_{t+1} \) and \( \rho_t \) are uniquely determined by \( k_{t+1} \) and \( k_t \) and \( \lambda(h_t) \) does not depend on \( e_{t+1} \). It follows that \( k_{t+1} \) depends on \( k_t \) and is not affected by the random variable \( e_{t+1} \).

Let us now derive the equilibrium value of \( k_{t+1} \) as a deterministic function of \( k_t \). Substituting (23) and (22) in (18), we obtain
\[ \int_0^{e_t} a^*_t(e_{t+1}; h_t) f(e_{t+1}) \, de_{t+1} = g^*_t(k_t(h_t))k_t(h_t). \] (26)

The market-clearing condition (8), together with eqs. (4) and (6), implies that the capital stock in period \( t + 1 \) is equal to the realized profits of the agent in \( t + 1 \), that is
\[ k_{t+1}(h_{t+1}) = \pi_{t+1}(h_{t+1}) = e_{t+1} + g_{t+1}(k_{t}(h_t)) - a^*_t(e_{t+1}; h_t). \] (27)

Solving (27) for \( a^*_t(e_{t+1}; h_t) \) and substituting in (26), we obtain
\[ k_{t+1}(e_{t+1}; h_t) = \bar{e} + g_{t+1}(k_{t}(h_t)) - g^*_t(k_t(h_t))k_t(h_t). \] (28)

Substituting now (28) into (27), given assumption (1), we obtain
\[ a^*_t(e_{t+1}; h_t) = (e_{t+1} - \bar{e}) + g^*_t(k_t(h_t))k_t(h_t) \geq 0. \] (29)

Eq. (29) and the market-clearing condition (8) imply that in equilibrium constraint (12) is always satisfied.

Proposition 2 is the consequence of the following facts:
(i) As expressed in Lemma 1, equilibrium in the asset market requires that prices adjust to insure a constant and uniform rate of return on all assets, independent of the stochastic shock \( \varepsilon \).
(ii) This rate of return represents the young agent's cost of acquiring a unit of capital. Given that capital is supplied inelastically by middle-aged agents, this cost must be equal to the expected marginal product of capital. Thus, as expressed by eq. (23), the amount of capital available in period \( t \) determines the rate of return on each asset.
(iii) At the same time, intertemporal optimization on the part of the representative agent implies that the rate of return on the asset he acquires in period \( t + 1 \) is equal to the shadow value of capital, discounted by the rate
of return prevailing in period $t$. It follows that the rate of return on each claim in period $t + 1$ is a function of the rate of return in period $t$, which is constant and independent of $\varepsilon_{t+1}$. Since in each period the rate of return on claims is uniquely determined by the stock of capital $k$, it follows that $k$ is a deterministic function of $k_{t+1}$.

Given Proposition 1, Proposition 2 tells us that the availability of contingent-claims markets eliminates an important source of inefficiency in the economy, represented by fluctuations in the stock of capital.

### 4. Standard loan contracts

Suppose now that while borrowers can observe the realization of $\varepsilon_t$ at no cost, lenders can only observe the realization of $\varepsilon_t$ at some cost that we call monitoring cost. In this case a moral-hazard problem arises and the market for contingent claims is no longer an optimal institutional arrangement. Since monitoring is costly, middle-aged agents will not be willing to observe the realization of $\varepsilon_t$ at all times; but whenever lenders decide not to monitor, there will be an incentive, for borrowers, not to reveal their information truthfully. In this case in fact, since the payment due to lenders is contingent on the realization of $\varepsilon_t$, young agents will always report higher realizations of $\varepsilon_t$ in order to decrease the payment to the lenders.

Let us now denote by $m_t$ the number of producers that are monitored by the same agent and let us assume that monitoring costs $\gamma_t$ are such that

\[
\gamma_t = \delta(m_t)(g_{t+1}(k_t) + \varepsilon_{t+1}),
\]

where $\delta'(m_t) < 0$ for $m_t < m$ and $\delta'(m_t) = 0$ for $m_t \geq m$. Monitoring costs are proportional to the sum of the output of the agent at the beginning of the following period and the exogenous endowment, while the proportionality factor decreases when the number of agents monitored increases, up to the point at which $m_t = \bar{m}$. Assumption (3.1) allows us to motivate the existence of 'financial intermediaries'. Since monitoring costs decrease when the number of agents monitored increases, there is an incentive for middle-aged agents to form 'coalitions' that collect funds from a number of producers not inferior to $\bar{m}$. In this model financial intermediaries, which arise in order to minimize monitoring costs, receive funds from middle-aged agents and lend them to young producers. In order to render the model closer to the reality of financial intermediation and to make the analysis simpler, we assume that the rate of return $i_t(h_t)$ intermediaries promise to pay on deposits at time $t$ is

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3In Diamond (1984), Gale and Hellwig (1985), and Williamson (1986, 1987b), financial intermediation arises because no agent is able to directly fund producers and in order to avoid the duplication of monitoring costs. This justification for the existence of financial intermediaries is not applicable to our context, since all the agents are identical and $\varepsilon$ is an aggregate shock.
independent of the realizations of the random variable $\varepsilon_{t+1}$ and is only a function of the history of the economy up to that time.\footnote{This assumption can be easily justified if we consider, as in Diamond (1984), Gale and Hellwig (1985), and Williamson (1986, 1987a, b), a situation in which depositors cannot observe the behavior of intermediaries. In this case, in fact, only a noncontingent rate of return eliminates the moral hazard problem that arises in the market for loanable funds. Notice, moreover, that it would not be feasible for intermediaries to make the return on deposits entirely contingent on the realizations of $\varepsilon$, given that intermediaries themselves do not always observe the shocks.} The consistency between this assumption and the equilibrium outcome of the model will be proved at the end of this section.

Let us now consider a situation in which an agent, in the second period of his life, has achieved a level of wealth $\pi_t(h_t)$ and has observed the realization of the random variable $\varepsilon_t$. The agent will acquire an amount of deposits from an intermediary such that

$$ \pi_t(h_t) = d_t. \quad (31) $$

Upon reaching the old age an agent receives the gross return from his investment $i_t(h_t)d_{t-1}$ and consumes all of it so that

$$ c_t = i_t(h_t)d_{t-1}. \quad (32) $$

In each moment in time, market clearing implies

$$ d_t = k_t. \quad (33) $$

Notice that given (31) and (33), the consumption of an agent in the third period of his life is completely determined by the level of wealth obtained in the first period of his life. Such level of wealth, in turn, depends on his production activity and on the contract signed with an intermediary. As in the complete-market case, we assume again that the agent maximizes the expected value of consumption in the third period of his life and, therefore, the agent acts as a risk-neutral, expected value maximizer. As in Diamond (1984), Gale and Hellwig (1985), and Williamson (1986, 1987a, b), the contract that arises optimally when monitoring is costly is a standard debt contract. Denoting by $\xi(\varepsilon_{t+1})$ the payment to the lender in period $t + 1$, an important property of such a contract is given by the following:

**Lemma 2.** When monitoring does not occur, $\xi(\varepsilon_{t+1})$ is independent of $\varepsilon_{t+1}$.\footnote{This assumption can be easily justified if we consider, as in Diamond (1984), Gale and Hellwig (1985), and Williamson (1986, 1987b), a situation in which depositors cannot observe the behavior of intermediaries. In this case, in fact, only a noncontingent rate of return eliminates the moral hazard problem that arises in the market for loanable funds. Notice, moreover, that it would not be feasible for intermediaries to make the return on deposits entirely contingent on the realizations of $\varepsilon$, given that intermediaries themselves do not always observe the shocks.}
as an interest factor, the fixed payment is given by the principal and the interest on the loan; when monitoring occurs the payment to a lender is contingent on the realization of \( e_{t+1} \). Once a contract defines the payment to the lender when monitoring occurs and when monitoring does not occur, we can immediately see that incentive compatibility requires that monitoring occur whenever \( R(e_{t+1}) < r_t k_t \), while there is no need for monitoring when \( R(e_{t+1}) \geq r_t k_t \). Lenders will monitor borrowers only when borrowers cannot make the payment and therefore the payment to the lender is again conditional on the realization of \( e_{t+1} \). When \( R(e_{t+1}) < r_t k_t \), we say that the borrower is bankrupt.

Letting \( A = \{ e_{t+1} : R(e_{t+1}) \geq r_t k_t \} \) and \( B = \{ e_{t+1} : R(e_{t+1}) < r_t k_t \} \), a borrower’s expected wealth at the end of the period is

\[
E_t \pi_{t+1}(h_{t+1}) = \int_A \left[ g_{t+1}(k_t) + e_{t+1} - r_t k_t \right] f(e_{t+1}) \, d\varepsilon_{t+1} + \int_B \left[ g_{t+1}(k_t) + e_{t+1} - R(e_{t+1}) \right] f(e_{t+1}) \, d\varepsilon_{t+1},
\]

\[(34)\]

i.e., the profits a borrower obtains if he repays the loan and monitoring does not occur and the profits a borrower obtains when monitoring occurs. Intermediaries’ expected profits from lending the amount \( k_t \) to an agent instead are

\[
E_t \rho(h_{t+1}) = r_t k_t \int_A f(e_{t+1}) \, d\varepsilon_{t+1} + \int_B \left[ R(e_{t+1}) - \delta(g_{t+1}(k_t) + e_{t+1}) \right] \times f(e_{t+1}) \, d\varepsilon_{t+1} - k_t i_t(h_t),
\]

\[(35)\]

i.e., the fixed repayment of the loan if the agent does not default, plus the state-contingent payment, minus the monitoring costs when the agent defaults, minus the expected cost of funds to intermediaries. Since we assume competition and free entry into the intermediation business, intermediaries’ profits, in equilibrium, are pushed to zero. Given eqs. (31)–(35), an optimal contract will be given by the levels of \( k_t \), \( r_t \), and \( R(e_{t+1}) \) that solve the following:

\[
\text{Problem 1. Maximize}
\]

\[
E_t e_{t+2},
\]

\[(36)\]

\[\text{Notice that in this model, instead of assuming price-taking behavior on the part of agents, we assume utility-taking behavior. The reason is that when contracts are incomplete and characterized by asymmetric information, the standard assumption that prices are set by a Walrasian auctioneer does not usually provide an appropriate characterization of competition. Walrasian equilibria, in fact, may be dominated by other equilibria where sellers (in our case, intermediaries) offer both prices and quantities.}\]
subject to
\[ k_{t+1} = g_{t+1}(k_{t}) + \varepsilon_{t+1} - r_t k_t \quad \text{if} \quad R(\varepsilon_{t+1}) \geq r_t k_t, \]  
\[ k_{t+1} = g_{t+1}(k_{t}) + \varepsilon_{t+1} - R(\varepsilon_{t+1}) \quad \text{if} \quad R(\varepsilon_{t+1}) < r_t k_t, \]  
\[ c_{t+2} = i_{t+1}(\varepsilon_{t+1}, h_t) k_{t+1}, \]  
\[ r_t k_t \int_A f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} + \int_B \left[ R(\varepsilon_{t+1}) - \delta g_{t+1}(k_{t}) + \varepsilon_{t+1} \right] \times f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} - k_t i_t(h_t) = 0, \]  
\[ c_{t+2} > 0. \]  

In order to obtain a solution to Problem 1 we will first show that, as in Gale and Hellwig (1985) and Williamson (1987a), an optimal contract requires that a creditor, in the event of bankruptcy, recovers as much as he can from a debtor, i.e., we prove:

**Lemma 3.** The solution to Problem 1 implies \( R(\varepsilon_{t+1}) = g_{t+1}(k_{t}) + \varepsilon_{t+1} \).

**Proof.** Denote by \( (R'(\varepsilon_{t+1}), r'_t, k'_t) \) the solution to Problem 1, by \( c'_{t+2} \) the corresponding level of consumption in period \( t + 2 \), and suppose that \( R'(\varepsilon_{t+1}) < g_{t+1}(k_{t}) + \varepsilon_{t+1} \). Let \( A' = \{ \varepsilon_{t+1} : R'(\varepsilon_{t+1}) \geq r'_t k'_t \} \) and \( B' = \{ \varepsilon_{t+1} : R'(\varepsilon_{t+1}) < r'_t k'_t \} \). Substituting (37), (38), and (39) into (36), we obtain

\[ c'_{t+2} - i_{t+1}(\varepsilon_{t+1}, h_t) \left[ g_{t+1}(k_{t}) + \varepsilon_{t+1} - \delta \int_{B'} (g_{t+1}(k_{t}) + \varepsilon_{t+1}) \times f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} - k_t i_t(h_t) \right] \]  

Since \( R'(\varepsilon_{t+1}) < g_{t+1}(k_{t}) + \varepsilon_{t+1} \) for some \( \varepsilon_{t+1} \in A' \), there exists another contract \( (R''(\varepsilon_{t+1}), r''_t, k''_t) \) such that (i) \( R''(\varepsilon_{t+1}) \geq R'(\varepsilon_{t+1}) \) for all \( \varepsilon_{t+1} \) and \( R''(\varepsilon_{t+1}) > R'(\varepsilon_{t+1}) \) for some \( \varepsilon_{t+1} \in A' \), with \( R''(\varepsilon_{t+1}) \) continuous and monotone increasing; (ii) \( 0 < r''_t < r'_t \), with \( A'' = \{ \varepsilon_{t+1} : R''(\varepsilon_{t+1}) > r''_t k''_t \} \) and \( B'' = \{ \varepsilon_{t+1} : R''(\varepsilon_{t+1}) < r''_t k''_t \} \) such that

\[ c''_{t+2} - c'_{t+2} = i_{t+1}(\varepsilon_{t+1}, h_t) \delta \left[ \int_{B'} (g_{t+1}(k_{t}) + \varepsilon_{t+1}) f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} \right. \]  
\[ - \left. \int_{B''} (g_{t+1}(k_{t}) + \varepsilon_{t+1}) f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} \right] > 0, \]  

which implies a contradiction. \( \square \)
This result is a consequence of the fact that with risk-neutral lenders and borrowers optimality requires that expected monitoring costs be minimized and this can happen only when the probability of bankruptcy is minimized. In turn, for this to occur, it is necessary that the penalty for bankruptcy is as high as possible. Eq. (41) tells us that if such penalty is not high enough, i.e., the payment due to the lender in case of default is lower than all the revenues of the borrower, it is possible to decrease the set of the states of nature under which bankruptcy occurs. By increasing the penalty for bankruptcy, therefore, it is possible to decrease the probability of default, increase the amount of capital that will be offered in the market and, consequently, future consumption. It follows that a contract where a firm must not surrender all of its revenues to lenders cannot be an equilibrium. Define now $\hat{\epsilon}_{t+1}$ as the level of $\epsilon_{t+1}$ under which producers are not able to repay their debt, i.e.,

$$\hat{\epsilon}_{t+1} = r_t k_t - g_{t+1}(k_t).$$  

(42)

Given (42), we see that $A = \{\epsilon_{t+1}: \epsilon_{t+1} \geq \hat{\epsilon}_{t+1}\}$ and $B = \{\epsilon_{t+1}: \epsilon_{t+1} < \hat{\epsilon}_{t+1}\}$. Substituting (42) into (37) and (39) and redefining the sets of integration, Problem 1 can be rewritten as the problem of choosing the levels of $r_t$ and $k_t$ that maximize (36).

**Problem 2.** Maximize

$$E_t c_{t+2},$$

subject to

$$
\begin{align*}
    k_{t+1} &= g_{t+1}(k_t) + \epsilon_{t+1} - r_t k_t & \text{if } \epsilon_{t+1} \geq \hat{\epsilon}_{t+1}, \\
    k_{t+1} &= 0 & \text{if } \epsilon_{t+1} < \hat{\epsilon}_{t+1},
\end{align*}
$$

(37')

$$c_{t+2} = i_{t+1}(\epsilon_{t+1}, h_t) k_{t+1},$$

(38)

$$r_t k_t \int_{\hat{\epsilon}_{t+1}}^{\epsilon_{t+1}} f(\epsilon_{t+1}) \, d\epsilon_{t+1} + (1 - \delta)$$

$$\times \int_0^{\hat{\epsilon}_{t+1}} (g_{t+1}(k_t) + \epsilon_{t+1}) f(\epsilon_{t+1}) \, d\epsilon_{t+1} - k_t i_t(h_t) = 0.$$  

(39')

$$c_{t+2} \geq 0.$$  

(40)
We can now prove:

**Proposition 3.** The solution to Problem 2 implies \( k_{t+1} = g_{t+1}(k_t) + \varepsilon_{t+1} - g_{t+1}'(k_t)k_t \).

**Proof.** We will first study Problem 2 ignoring constraint (40); we will then show that the solution to such a problem always satisfies constraint (40). Substituting (37'), (38), and (39') into (36), Problem 2 can be rewritten as the problem of choosing the level of \( r_t \) and \( k_t \) that maximize

\[
\int_{\hat{e}_{t+1}}^{\bar{e}_{t+1}} (g_{t+1}(k_t) + \varepsilon_{t+1} - r_t k_t) f(\varepsilon_{t+1}) \, d\varepsilon_{t+1},
\]

subject to

\[
r_t k_t \int_{\hat{e}_{t+1}}^{\bar{e}_{t+1}} f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} + (1 - \delta) \times \int_0^{\hat{e}_{t+1}} (g_{t+1}(k_t) + \varepsilon_{t+1}) f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} - k_t h_t = 0.
\]

Notice that eq. (39') implicitly defines a function \( r_t(k_t) \). Substituting such a function into (43), the problem becomes that of choosing the level of \( k_t \) that maximizes

\[
\int_{\hat{e}_{t+1}}^{\bar{e}_{t+1}} (g_{t+1}(k_t) + \varepsilon_{t+1} - r_t(k_t)k_t) f(\varepsilon_{t+1}) \, d\varepsilon_{t+1}.
\]

The first-order conditions imply

\[
g_{t+1}'(k_t) \left[ (1 - F(\hat{\varepsilon}_{t+1})) + (1 - \delta) F(\hat{\varepsilon}_{t+1}) \right] = i_t(h_t).
\]

Substituting now (45) into (39'), we obtain

\[
r_t k_t = g_{t+1}'(k_t)k_t - \left[ (1 - \delta) / (1 - F(\hat{\varepsilon}_{t+1})) \right] \times \int_0^{\hat{e}_{t+1}} (g_{t+1}(k_t) + \varepsilon_{t+1} - g_{t+1}'(k_t)k_t) f(\varepsilon_{t+1}) \, d\varepsilon_{t+1}.
\]
Substituting in turn (46) into (37'), we get

\[ k_{t+1} = g_{t+1}(k_t) + \varepsilon_{t+1} - g'_{t+1}(k_t)k_t - \left[ (1 - \delta) / (1 - F(\hat{\varepsilon}_{t+1})) \right] \]

\[ \times \int_{\hat{\varepsilon}_{t+1}}^{\varepsilon_{t+1}} (g_{t+1}(k_t) + \varepsilon_{t+1} - g'_{t+1}(k_t)k_t)f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} \]

(47)

\[ if \quad \varepsilon_{t+1} \geq \hat{\varepsilon}_{t+1}, \]

\[ k_{t+1} = 0 \quad if \quad \varepsilon_{t+1} < \hat{\varepsilon}_{t+1}. \]

Given (46), we also have

\[ \hat{\varepsilon}_{t+1} = - \left[ (1 - \delta) / (1 - F(\hat{\varepsilon}_{t+1})) \right] \]

\[ \times \int_{\hat{\varepsilon}_{t+1}}^{\varepsilon_{t+1}} (g_{t+1}(k_t) + \varepsilon_{t+1} - g'_{t+1}(k_t)k_t)f(\varepsilon_{t+1}) \, d\varepsilon_{t+1} \]

\[ - [g_{t+1}(k_t) - g'_{t+1}(k_t)k_t] < 0, \]

(48)

which implies that producers will never default on their loans. Given (48), eq. (46) reduces to

\[ k_{t+1} = g_{t+1}(k_t) + \varepsilon_{t+1} - g'_{t+1}(k_t)k_t. \]

(49)

Notice also that (39') and (48) in period \( t + 1 \) imply

\[ g'_{t+1}(k_t) = i_t(h_t), \]

(50)

which, given (38), implies that constraint (40) is always satisfied.

The result of Proposition 2 gives us the dynamics of the model and tells us that, in each moment in time when standard debt contracts prevail because of asymmetric information and monitoring costs, the level of capital depends on the realizations of the stochastic variable \( \varepsilon \). This is a consequence of the fact that young agents' income represents the capital stock available in the following period to the next generations. Since standard loan contracts imply the payment of a predetermined interest rate and the repayment of the principal, the agent does not have any possibility of insuring his income against stochastic fluctuations in the endowment. Standard loan contracts do not allow agents to provide a deterministic level of capital and, therefore, given Proposition 1, they impose a serious inefficiency on the economy.
Denoting by $\rho_{t+1}$ the actual profits of an intermediary in period $t$, since $\hat{\rho}_{t+1} < 0$, we notice that

$$\rho_{t+1} = g_{t+1}(k_t)k_t - k_t\hat{\rho}_t(h_t) = 0,$$

which implies that intermediaries will always be able to pay a deterministic rate of return on deposits. In this model, therefore, the assumption of a noncontingent interest rate on loanable funds is fully consistent with equilibrium.

5. Capital accumulation under different financial agreements:
A comparison

In the previous sections, we derived a deterministic difference equation that defines the stock of capital at time $t + 1$ when contingent-claims markets exist (hereinafter referred to as $k^c_{t+1}$) and a stochastic difference equation that defines the level of capital at time $t + 1$ when standard loan contracts prevail (hereinafter referred to as $k^l_{t+1}$).

In this section, we will compare $k^c_{t+1}$ (given any initial condition $k_0$) with the expected value of $k^l_{t+1}$, $E(k^l_{t+1} | k_0)$. Therefore, we establish:

**Proposition 4.** If, for every $k > 0$, $-kg^m_{t+1}(k)/g^m_{t+1}(k) < 1$, then, for any initial $k_0$, $k^c_{t+1} > E(k^l_{t+1} | k_0)$.

**Proof.** Let us first define $\gamma_t(k_t) = g_{t+1}(k_t) - k_tg^*_t(k_t)$ and express eq. (28) as

$$k^c_{t+1} = \bar{\epsilon} + \gamma_t(k_t).$$

Notice that $\gamma_t(k_t) = -g^m_{t+1}(k_t)/g^m_{t+1}(k_t) > 0$ and, by assumption, $\gamma_t = -g^m_{t+1}(k_t) - k_tg^m_{t+1}(k_t) < 0$. Since, in turn, $\bar{k}_t = \bar{\epsilon} + \gamma_{t-1}(k_{t-1})$, we can express (52) as

$$k^c_{t+1} = \bar{\epsilon} + \gamma_t(\bar{\epsilon} + \gamma_{t-1}(k_{t-1})).$$

Substituting again for $k_{t-2}, k_{t-3}, \ldots, k_0$, we obtain

$$k^c_{t+1} = \bar{\epsilon} + \gamma_t(\bar{\epsilon} + \gamma_{t-1}(\bar{\epsilon} + \gamma_{t-2}(\ldots \bar{\epsilon} + \gamma_0(k_0))))).$$

Notice now that

$$E(k^l_{t+1} | k_t) = \bar{\epsilon} + E(g_{t+1}(k_t) - k_t\hat{\rho}_t(h_t)) = \bar{\epsilon} + E\gamma_t(k_t).$$

Since $\gamma_t(k_t)$ is concave, Jensen’s inequality implies

$$E(k^l_{t+1} | k_t) = \bar{\epsilon} + E\gamma_t(k_t) < \bar{\epsilon} + \gamma_t(Ek_t).$$
Similarly,
\[ E(k_t|k_{t-1}) = \bar{\varepsilon} + E\gamma_{t-1}(k_{t-1}) < \bar{\varepsilon} + \gamma_{t-1}(Ek_{t-1}), \] (57)
and therefore,
\[ E(k_{t+1}|k_t) < \bar{\varepsilon} + \gamma_t(\bar{\varepsilon} + \gamma_{t-1}(Ek_{t-1})). \] (58)
Substituting again for \( Ek_{t-1}, Ek_{t-2}, \ldots, Ek_0 \) and using again Jensen’s inequality, we obtain
\[ E(k_{t+1}|k_t) < \bar{\varepsilon} + \gamma_t(\bar{\varepsilon} + \gamma_{t-1}(\ldots + \gamma_0(k_0)))) = k^c_{t+1}. \] (59)

When standard loan contracts prevail, therefore, the expected stock of capital in each period is smaller than when contingent-claims markets are available, provided that \(-kg''_{t+1}(k)/g''_{t+1}(k) < 1\). Notice that this condition is satisfied, for example, by the isoelastic production function.

6. Conclusions

We have analyzed the model of a simple economy where different types of financial agreements produce different effects on capital accumulation. The main result of our study is the finding that, when contingent-claims markets exist, the growth of the capital stock is independent of the random shocks to which the economy is subject. When instead contingent-claims markets are absent and standard loan contracts prevail, the economy will incur an efficiency loss, represented by undesirable fluctuations in the level of capital. Moreover, the level of capital achieved under contingent-claims markets is always greater than the expected level of capital that prevails when only standard loan contracts are available.

References

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