Optimal monetary policy in economies with dual labor markets

Fabrizio Mattesini \(^{a}\), Lorenza Rossi \(^{b, *}\)

\(^{a}\) Università di Roma “Tor Vergata” Via Columbia 2, 00133 Rome, Italy
\(^{b}\) University of Pavia, Department of Economics and Quantitative Methods, Via San Felice al Monastero 5, 27100 Pavia, Italy

**Abstract**

We present a dynamic stochastic general equilibrium (DSGE) New Keynesian model with indivisible labor and a dual labor market: a Walrasian one where wages are fully flexible and a unionized one characterized by real wage rigidity. We show that the negative effect of a productivity shock on inflation and the positive effect of a cost-push shock are crucially determined by the proportion of firms that belong to the unionized sector. The larger this number, the larger are these effects. Consequently, the larger the union coverage, the larger should be the optimal response of the nominal interest rate to exogenous productivity and cost-push shocks. The optimal inflation and output gap volatility increases as the number of the unionized firms in the economy increases.

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1. Introduction

One of the most striking differences among modern industrialized economies is the role trade unions play in determining wages and employment conditions. While in the US only about 15% of workers are covered by collective contract agreements, in the UK this percentage is about 36% and in countries such as France, Italy or Sweden is much higher, rising above 84%.\(^{1}\) Given the importance of labor markets in determining output, inflation and the response of the economy to aggregate shocks, a very natural question is whether and how central banks, in formulating monetary policy, should take into account the structure of industrial relationships.

In this paper we address this issue by studying optimal monetary policy in a dynamic stochastic general equilibrium New Keynesian (DSGE-NK henceforth) model\(^{2}\) with a dual labor market. Firms may belong to two different final-goods-producing sectors: one where wages and employment are determined under perfect competition, and the other where wages and employment are the result of a contractual process between unions and firms. As in Hansen (1985) and Rogerson and Wright (1988), labor supply is indivisible and workers face a positive probability to remain unemployed.

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\(^{*}\) Corresponding author.

E-mail address: lorenza.rossi@eco.unipv.it (L. Rossi).

\(^{1}\) More precisely, the number of persons covered by collective agreements over total employment was 94.5% in France in 2003, 84.1% in Italy in the year 2000 and 85.1% in Sweden in the year 2000. The data about US and UK refer to the year 2002. For a complete set of data on union coverage on the various countries see Lawrence and Ishikawa (2005).

\(^{2}\) The only paper that explicitly consider the role of trade unions in a DSGE-NK model is the one by Zanetti (2007) who, however, does not focus on normative aspects and studies separately economies characterized by monopolistic unions and economies characterized by competitive labor markets.

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Wages in the unionized sector are set according to the popular monopoly-union model developed by Dunlop (1944) and Oswald (1982) which has been recently introduced in a real business cycle (RBC) model by Maffezzoli (2001) and in a DSGE-NK model by Zanetti (2007).

By doing this we depart from the recent literature, that has recently analyzed search and matching frictions à la Mortensen and Pissarides (1994) in DSGE-NK models and we concentrate on the consequences of collective bargaining between unions and firms. Unions, in this model, do not simply maximize the utility of their members, but are institutions that also have “political” objectives in the sense that their objective function takes into account the preferences of workers, the preferences of leaders and market constraints. In this respect we take side on the old and never settled debate initiated by Dunlop (1944) and Ross (1948) over the appropriate maximand for the unions’ utility function, and we assume that the unions’ objective function is a Stone-Geary utility function as in Dertouzos and Pencavel (1980), Pencavel (1984) and, more recently, in De la Croix et al. (1996), Raurich and Sorolla (2003) and in Chang et al. (2007). This function is extremely flexible and, depending on parameter values, allows for different distribution of power, inside the union, between members and leaders who may have diverging objectives. The divergence between the union’s objective and households’ utility creates a distortion in the economy and gives rise to real wage rigidity. Interestingly, wage rigidity does not apply only to new hirings, as in the model with search and matching frictions (see for example Thomas, 2008) but also to ongoing relationships.

The presence of a unionized sector has very important consequences for monetary policy. What Blanchard and Galí (2007), define as the “divine coincidence” does not generally hold: for a central bank stabilizing output around the level that would prevail under flexible prices (natural output) is not equivalent to pursuing the efficient level of output and a trade-off arises between output stabilization and inflation stabilization. A first major result of our model is that the trade-off between inflation stabilization and the level of output (and unemployment) depends on the relative weight of the unionized and competitive sectors: the larger is the fraction of firms that are able to set wages in a unionized labor market, the larger is the trade-off they face in response to productivity shocks. This has significant consequences for optimal monetary policy that we derive, as in Woodford (2003), from the maximization by the central bank of a second-order approximation of agents’ utility function.

We find that, differently from the standard New Keynesian model where monetary policy must not respond to technology shocks, in our model monetary policy must be procyclical in response to such shocks. Moreover, and this is the second major result of this paper, monetary policy must be progressively more accommodating as the size of the unionized sector increases; in an economy where labor markets are mainly competitive, the nominal interest rate must decrease much less in response to a productivity shock than in an economy where wages are largely set by collective bargaining between unions and firms.

The procyclicality of optimal monetary policy and its dependence on union coverage represent a significant departure from the most recent contributions such as Faia (2008) where optimal monetary policy is procyclical only for some parameters of the matching technology and Blanchard and Galí (2008) where the main friction characterizing labor markets are hiring costs. In our model, if we consider two countries hit by the same shocks and where the central bank behaves optimally, we observe that in the country where the number of “Walrasian” firms is larger, the interest rate will vary much less than in the other country. This, however, is not the consequence of differences in the reaction functions of the two central banks to a unit change in expected inflation; rather it is caused by the fact that the economy where the labor market is more competitive experiences smaller inflationary tensions.

Our model provides also a convenient framework to address important normative issues such as, for example, the optimal behavior of central banks in periods characterized by labor market turmoil and exogenous wage shocks. In the framework we propose here a policy trade-off for the central bank arises also in response to exogenous changes in the unions’ reservation wage, that we interpret as cost-push shocks. If the unions’ reservation wage is subject to exogenous changes, and these changes tend to be persistent over time, then a welfare maximizing central bank must again face the problem of whether to accommodate these shocks with a easier monetary policy. As in the case of technology shocks, also in this case optimal monetary policy requires only partial accommodation, and the response of the central bank is crucially determined by the fraction of firms that, in the economy, set wages competitively.

We finally calibrate the model and we analyze the differences between an economy where the central bank follows a standard Taylor rule, as the one estimated by Smets and Wouters (2003) for Europe, and an economy where the central bank follows the optimal rule. The calibration of the model under the Taylor rule estimated by Smets and Wouters (2003) for Europe shows that our model is able to qualitatively replicate the dynamics of the main economic variables and that a unionized economy tends to have larger responses to productivity shocks than an economy where competitive labor markets prevail. The difference in the impulse response functions (IRFs) between these two types of economies becomes

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4 Faia (2008), in a model with search and matching frictions, shows that optimal monetary policy should be procyclical only when worker’s bargaining power is higher than the share of unemployed people in the matching technology. Blanchard and Galí (2008) study instead optimal monetary policy in an environment characterized by hiring costs and real wage rigidity and also show that in countries with more “sclerotic” labor markets monetary policy should be more accommodating than the one that should be pursued in more flexible.
much larger, however, under an optimal monetary policy. Optimality implies also that monetary policy be much more accommodating when wages are the result of bargaining between unions and firms.

The paper is organized as follows. In Section 2 we develop a DSGE-NK model with indivisible labor and two-sector labor market while in Section 3 we study optimal monetary policy and the optimal volatility of inflation and output gap. In Section 4 we discuss the calibration of the model under the optimal policy with different degrees of the union coverage. Section 5 concludes.

2. The model

2.1. The representative household

We consider an economy populated by many identical, infinitely lived worker-households each of measure zero. Households demand a Dixit and Stiglitz (1977) composite consumption bundle produced by a continuum of monopolistically competitive firms. In each period households sell labor services to the firms and each firm is endowed with a pool of households from which it can hire. As a matter of fact firms hire workers from a pool composed of infinitely many households so that the individual household member is again of measure zero.

We assume that the economy is composed of two sectors \( u \) and \( w \) that produce two different consumption bundles. In sector \( u \) workers are represented by a union that tries to extract some producer surplus from firms. In sector \( w \) the labor market is competitive.\(^5\) The number of firms in the competitive sector is \( q \) while the number of firms in the unionized sector is \( 1 - q \). Labor is homogeneous and workers are assigned randomly to each sector. Given the structure of the economy, \( q \) not only represents the number of firms that face a Walrasian labor market but also the probability that a worker is assigned to the Walrasian sector. Once a household is assigned to a firm specific sector, as in Hansen (1985), Rogerson (1988) and Rogerson and Wright (1988), it has the alternative between working a fixed number of hours and not working at all. For the sake of simplicity we assume that \( q \) is constant.

Let us first consider the problem of an agent that supplies his labor to a firm in the Walrasian sector, i.e. to a firm that faces a competitive labor market where firms and workers act as price takers. We assume that households enter employment lotteries, i.e. sign with a firm a contract that commits them to work a fixed number of hours, that we normalize to one, with probability \( q \). Labor is homogeneous and workers are assigned randomly to each sector. Given the structure of the economy, \( q \) not only represents the number of firms that face a Walrasian labor market but also the probability that a worker is assigned to the Walrasian sector. Once a household is assigned to a firm specific sector, as in Hansen (1985), Rogerson (1988) and Rogerson and Wright (1988), it has the alternative between working a fixed number of hours and not working at all. For the sake of simplicity we assume that \( q \) is constant.

Let us now consider the case of a household that works in a unionized labor market. The unionized sector is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiates with a single union \( i \in (0, 1) \), which is too small to influence the outcome of the market. Unions negotiate the wage on behalf of their members. Once the wage rate is defined, firms chose the amount of labor that maximizes their profits. Similarly to what happens in the competitive case, labor is indivisible and workers participate to employment lotteries. As in the previous case, therefore, before the lottery draw, the expected intratemporal utility function of workers, who happens to belong to the unionized sector is

\[
N^u_i[C^u_{it}u(0)]^{1-\sigma} + (1 - N^w_i)[C^w_{it}u(1)]^{1-\sigma}
\]

where \( C^u_{it} \) is the consumption level of employed individuals. We denote by \( u(\cdot) \) the utility of leisure. Since the utility of leisure of employed individuals \( u(0) \) and the utility of leisure of unemployed individuals \( u(1) \) are positive constants, we assume \( u(0) = u_0 \) and \( u(1) = u_1 \). As in King and Rebelo (1999),\(^6\) we assume \( u_0 < u_1 \).

Since they face a probability \( 1 - N^w_i \) of not working at all, workers will try to acquire insurance against the risk of remaining unemployed. We assume that asset markets are complete, so that employed and unemployed individuals are able to achieve perfect risk sharing, equating the marginal utility of consumption across states.

Let us now consider the case of a household that works in a unionized labor market. The unionized sector is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiates with a single union \( i \in (0, 1) \), which is too small to influence the outcome of the market. Unions negotiate the wage on behalf of their members. Once the wage rate is defined, firms chose the amount of labor that maximizes their profits. Similarly to what happens in the competitive case, labor is indivisible and workers participate to employment lotteries. As in the previous case, therefore, before the lottery draw, the expected intratemporal utility function of workers, who happens to belong to the unionized sector is

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N^u_i[C^u_{it}u(0)]^{1-\sigma} + (1 - N^w_i)[C^u_{it}u(1)]^{1-\sigma}
\]

where \( C^u_{it} \) is the consumption level of employed individuals and \( N^u_i \) is the probability of being employed for agents belonging to the unionized sector. Again, we assume \( u(0) = u_0 \) and \( u(1) = u_1 \). Since they face a positive probability of being unemployed, risk averse workers will try to obtain insurance against the risk of being unemployed; access to complete asset markets will allow individuals to achieve perfect risk sharing.

It is important to observe that, beside the risk of remaining unemployed, workers in this model face also another type of uncertainty since they do not know, a priori, whether they will participate to a competitive labor market or to a unionized

\(^5\) The idea behind this model is that in some sectors of the economy the cost of forming a union is high while in other sectors is lower. However, we do not analyze, in this paper, the process of union formation but simply assume that in one of the two sector unions already exist. When assigned to this sector, workers can contract with firms only through the union.

\(^6\) This depends on the fact that the utility of leisure \( \phi(1 - N_i) \) as usual, is an increasing function of the time spend in leisure. Given that the time spend in leisure is greater for unemployed agent than for employed agent this means that \( u(1) > u(0) \).
one. We assume that, through complete asset markets, agents can also acquire insurance against the income fluctuations implied by this type of uncertainty. As we show in Appendix A, with this structure we are able to write the life-time expected intertemporal utility function of a representative household as

\[
U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left( \frac{1}{1-\beta} \right) [C_t \phi(N_t)]^{1-\alpha}
\]  

(3)

where \(0 < \beta < 1\) is the subjective discount rate. We define

\[
\phi(N_t) = \frac{(qN_t^w + (1 - q)N_t^u)\mu_0^{(1-\sigma)/\sigma} + (qN_t^w + (1 - q)N_t^u)\mu_1^{(1-\sigma)/\sigma}}{(1 - q)\mu_1^{(1-\sigma)/\sigma} + (1 - q)\mu_1^{(1-\sigma)/\sigma}}
\]

(4)

where \(N_t = qN_t^w + (1 - q)N_t^u\) is the probability to be employed. The flow budget constraint of the representative household is given by

\[
P_tC_t + R_t^{-1}B_t \leq qW_t^wN_t^w + (1 - q)W_t^uN_t^u + B_{t-1} + \Pi_t - T_t
\]

(5)

where \(W_t^h (h = w, u)\) is the wage rate in the two sectors. Total consumption \(C_t\) is a geometric average of consumption of the good produced in the Walrasian sector, \(C_{w,t}\), and of the good produced in the unionized sector, \(C_{u,t}\). Then,

\[
C_t = \left( \frac{C_{w,t}}{q^0(1-q)^{1-q}} \right)
\]

(6)

is the corresponding consumption price index (CPI) which is derived in Appendix C, and \(P_t^w\) and \(P_t^u\) are, respectively, the price index of goods produced in the Walrasian and the unionized sectors. The purchase of consumption goods is financed by labor income, profit income \(\Pi_t\), and a lump-sum transfers \(T_t\) from the Government. We assume that agents can also have access to a financial market where nominal bonds are exchanged. We denote by \(B_t\) the holdings of a nominal bond carried over from period \(t\) that pays one unit of currency in period \(t + 1\). Its price is \(R_t^{-1}\), where \(R_t\) denotes the gross nominal yield.\(^7\)

In solving the maximization of (3) subject to (4) we should remember that the worker chooses the levels of consumption \(C_t\) and \(C_{t+1}\) and the supply of labor \(N_t^w\), while \(N_t^u\) is taken as given, as it is determined by the union together with the firm. The first-order conditions imply

\[
1 = \beta R_t E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{\phi(N_t)}{\phi(N_{t+1})} \right)^{1-\sigma} \frac{P_t}{P_{t+1}} \right]
\]

(7)

\[
\frac{W_t^w}{P_t} = \frac{C_t^{1-\sigma} \phi(N_t) \phi(N_t)}{\phi(N_{t+1})} = -C_t^{\sigma} \frac{\phi(N_t)}{\phi(N_t)}
\]

(8)

where Eq. (7) is the standard consumption Euler equation.\(^8\) Eq. (8) holds only for households employed in the Walrasian sector. Optimality requires that the no-Ponzi game condition on wealth is also satisfied.

2.2. Final goods-producing firms

In sector \(w\) a perfectly competitive final good producer purchases \(Y_w^j(j)\) units of each intermediate good \(j \in [0, q]\) at a nominal price \(P_w^j(j)\) to produce \(Y_w^j\) units of the final good \(w\), while in sector \(u\) a perfectly competitive final good producer purchases a \(Y_u^j(j)\) units of each intermediate good \(j \in [q, 1]\) at a nominal price \(P_u^j(j)\) to produce \(Y_u^j\) units of the final good \(u\). Final goods-producing firms use the following constant returns to scale technology:

\[
Y_w^j = \left( \frac{1}{q} \right) \frac{\partial}{\partial j} Y_w^j(j)(0-1)/d_j \right]^{1/(0-1)}
\]

(9)

Profit maximization yields the following set of demands for intermediate goods:

\[
Y_t^w(j) = \frac{1}{q} P_t^w(j) Y_t^w \quad \text{and} \quad Y_t^u(j) = \frac{1}{1-q} \left( P_t^u(j) \right)^{-\theta} Y_t^u
\]

(10)

\(^7\) As is standard in New Keynesian models, government bonds are introduced here as a simple way to allow for the existence of a nominal interest rate in the economy, which will be the policy instrument of the Central Bank.

\(^8\) See Appendix B for derivation.
where \( \theta > 1 \) is the elasticity of substitution across intermediate goods, which is equal for the two sectors. In Appendix B we show that

\[
P^w_t = \left[ \int_0^q \frac{1}{q} P^w_t (j)^{1-\theta} \, dj \right]^{1/(1-\theta)} \quad \text{and} \quad P^u_t = \left[ \int_q^1 \frac{1}{1-q} P^u_t (j)^{1-\theta} \, dj \right]^{1/(1-\theta)}
\]

are the price indexes of the Walrasian and unionized sectors.

2.3. Intermediate goods-producing firms

We abstract from capital accumulation and assume that the representative intermediate good-producing firm \( h = w, u \), hires \( N^h_t \) units of labor from the household and produces \( Y^h_t (j) \) units of the intermediate good using the following technology:

\[
Y^h_t (j) = A_t N^h_t (j)^2
\]

where \( A_t \) is an exogenous productivity shock common to all firms. We assume that \( \ln A_t \equiv a_t \) follows the autoregressive process:

\[
a_t = \rho \alpha a_{t-1} + \delta_t
\]

where \( \rho < 1 \) and \( \delta_t \) is a normally distributed serially uncorrelated innovation with zero mean and standard deviation \( \sigma_\alpha \). The assumption of decreasing returns to scale, which is in line with a non-competitive intermediate good sector, has important implications on the optimal price-setting rule, and then on the derivation of the traditional Phillips curve.\(^9\)

Before choosing the price of its goods, a firm chooses the level of \( N^h_t (j) \) which minimizes its total costs, obtaining the following labor demand:

\[
(1 - \tau^h) \frac{W^h_t (j)}{P^h_t (j)} = \frac{MC^{h,j}_t}{P^h_t (j)} = \frac{Y^h_t (j)}{N^h_t (j)}
\]

where \( MC^{h,j}_t \) represent the nominal marginal costs of firm \( j \) in sector \( h \) and where \( \tau^h \) represents an employment subsidy to the sector \( h \) firm, which is set so that the steady state equilibrium in both sectors coincides with the efficient one.\(^10\)

Aggregating across firms \( j \), sector \( h \) real marginal costs are

\[
\frac{MC^{h,j}_t}{P^h_t} = \frac{(1 - \tau^h) W^h_t N^h_t}{P^h_t Y^h_t}
\]

2.4. Unions’ wage setting

For households hired by firms in the unionized sector, unions negotiate wages on behalf of their members. Since each household supplies its labor to only one firm, which can be clearly identified, workers try to extract some producer surplus by organizing themselves into a firm-specific trade union. The economy is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiates with a single union \( i \in (0,1) \) which is too small to influence the market.\(^11\) Unions negotiate the wage on behalf of their members.

Once unions are introduced in the analysis, two important issues arise: what is the objective function of the union and what are the variables of the bargaining process. Both these questions have been extensively investigated by the literature, although no conclusive agreement has been reached on the issue.\(^12\) The problem of identifying an appropriate maximand for the union dates back to Dunlop (1944) and Ross (1948); since then the debate has revolved over the relative importance of economic considerations (basically how employers respond to wage bargaining) and political considerations in the determination of union wage policy. For political considerations we intend how the preferences of workers, the preferences of union leaders and market constraints interact in determining a union’s objective.

One approach often followed in the literature is the “utilitarian” approach pioneered by Oswald (1982) which consists on assuming that all workers are equal and that the union simply maximizes the sum of workers’ utility, defined over wages. Although simple and appealing because coherent with a standard economic approach, this formulation of unions’ utility does not allow for political considerations. An alternative approach, initially proposed by Dertouzos and Pencavel (1980) and Pencavel (1984) and, more recently, reproposed by De la Croix et al. (1996) and Raurich and Sorolla (2003), is to

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\(^9\) In fact, as shown in Shbdone (2002) and in Galí et al. (2001), it should be taken into account that marginal costs are no longer common across firms.

\(^10\) We assume that the subsidy is covered by a lump-sum tax in that the Government runs always a balanced budget.

\(^11\) For tractability, we consider atomistic unions and we abstract in this paper from the issue of strategic interaction between unions and central banks.

\(^12\) For an extensive survey of unions model see Farber (1986), and, more recently, Kaufman (2002).
assume that unions maximize a modified Stone-Geary utility function of the form

\[ V\left(\frac{W_r^u}{P_t}, N_r^u\right) = \left(\frac{W_r^u(i)}{P_t} - \frac{W_r^u}{P_t}\right)^\gamma N_r^u(i)^\alpha \]  

(16)

The relative value of \(\gamma\) and \(\zeta\) is an indicator of the relative importance of wages and employment in the union's objective function.\(^{14}\) The reservation wage \(W_r^u(i)\) is the absolute minimum wage the union \(i\) can tolerate. This reservation wage has many possible interpretations. One possible interpretation is that \(W_r^u\) is the opportunity wage of the workers (Pencavel, 1984) since it is unlikely that a union can survive if it negotiates a wage below such level. Another possible interpretation is that \(W_r^u\) is what Blanchard and Katz (1999) define as an "aspiration wage", i.e. a wage that workers have come to regard as "fair". Unions' reservation wage is generally unobservable and therefore hard to model.\(^{15}\) Similarly to De la Croix et al. (1996), however, we assume that

\[ \frac{W_r^u(i)}{P_t} = \zeta w^u_t \]  

(17)

where \(\zeta > 0\) is a positive constant (which can be normalized to 1)\(^{16}\) and where

\[ w^u_t = \rho w_{t-1}^u + \xi^w_t \]  

(18)

with \(\rho < 1\) and \(\xi^w_t\) equal to a normally distributed serially uncorrelated innovation with zero mean and standard deviation \(\sigma_w\). If the real reservation wage is constant, \(\xi^w_t = 0\). Moreover, the fact that the reservation wage is subject to persistent shocks is meant to capture the exogenous wage shocks, often associated with political and social factors that have often characterized industrialized economies, especially in Europe.\(^{17}\)

The Stone-Geary utility function not only is appealing, both for its ability to approximate the actual behavior of unions and for its flexibility and tractability, but also for its generality. The parameters \(\gamma\) and \(\zeta\) correspond to the elasticities of the union's objective \(V\) to the excess wage \(W_r^u(i)/P_t - W_r^u(i)/P_t\) and to the employment level \(N_r^u(i)\), respectively. The larger the difference \(\zeta - \gamma\), the more the union approaches the extreme of a "democratic" (or "populist") union. When \(\zeta = \gamma\), these two parties have an identical discretionary power in formulating policies. If unions are "wage oriented" then \(\gamma > \zeta\), on the other hand if they are "employment oriented" \(\gamma < \zeta\). If we set \(\gamma = 1\), \(\zeta = 1\), maximizing (16) is equivalent to maximize the unions' objective function assumed by Maffezzoli (2001) and Zanetti (2007) in their recent papers. The bargaining process we consider here is in the tradition of the "right to manage" models. In particular, we follow the popular monopoly union model first proposed by (1984) and Oswald (1982), where the employment rate and the wage rate are determined in a non-cooperative dynamic game between unions and firms. We restrict the attention to Markov strategies, so that in each period unions and firms solve a sequence of independent static games. Each union behaves as a Stackelberg leader and each firm as a Stackelberg follower. Once the wage has been chosen, each firm decides the employment rate along its labor demand function. Even if unions are large at the firm level, they are small at the economy level, and therefore they take the aggregate wage as given. The ex ante probability of being employed is equal to the aggregate employment rate and the allocation of union members to work or leisure is completely random and independent over time. Finally, as in the Walrasian market, we assume that workers are able to perfectly insure themselves against the possibility of being unemployed. This result can either be obtained through the lottery mechanism previously described or by assuming, as in Maffezzoli (2001) and Zanetti (2007) that, in order to impede workers from leaving the union, the union pursues a redistributive goal, acting as a substitute for competitive insurance market. Insurance is supplied under zero-profit condition and is therefore actuarially fair. The cost minimization problem of the firm is the same as in the Walrasian market.

From the first-order conditions of the union's maximization problem with respect to the real wage \(W_r^u(i)/P_t\), after imposing the symmetric equilibrium we obtain

\[ \frac{W_r^u}{P_t} = \frac{\zeta}{\zeta - \gamma(1 - \alpha)} \frac{W_r^u}{P_t} \]  

(19)

\(^{13}\) As it can be easily verified, if unions set wage to simply maximize agents' utility, the wage schedule would be similar to the labor supply in the indivisible labor model, with the difference that the wage would be a constant mark-up over the marginal rate of substitution. In this case, wages would fully respond to technology shocks and no significant trade-off between inflation and unemployment (output gap) would emerge. Therefore, assuming that the union leader has this type of objective function is a very simple and realistic way to obtain endogenous real wage rigidities.

\(^{14}\) Our objective function is closed to the one suggested by the Ross (1948) tradition. In fact, for different parameter values, the union's objective function is almost equivalent to the one of a union which maximizes his income or his membership, as for example in Skatun (1995) and in Booth (1984).

\(^{15}\) One possibility is that the real reservation wage is tied to the competitive wage. Since, however, in this model we assume that workers, once matched to the sector, must work in that sector for one period there is no reason for considering the real reservation wage is tied to the competitive wage. In line with the "political" interpretation of the union behavior we prefer to model the real reservation wage as exogenous.

\(^{16}\) Alternatively we can assume that \(\zeta\) coincides with the steady state value of the wage obtained by workers employed in the Walrasian labor market. In this way the reservation wage can be viewed as the unions' threat point.

\(^{17}\) We consider both these two alternative in order to show that our results on the endogenous inflation unemployment (output) trade-off is not qualitatively influenced by the fact that the reservation wage shock is an exogenous shock.
with $\zeta > \gamma (1 - \alpha)$. The technology shock has no effect on the real wage rate chosen by the monopoly union. Since $\zeta / (\zeta - \gamma (1 - \alpha)) > 1$, we see that the real wage rate is always set above the reservation wage.

It is interesting to compare, at this point, Eq. (19) with the real wage equation we would obtain if the union simply maximized agents’ utility (3) subject to a firm’s labor demand (14). In this case the real wage would be given by

$$\frac{W_t^u}{P_t} = -\frac{1}{\alpha} C_t \frac{\phi_u(N_t)}{\phi(N_t)}$$

(20)

where $1 / \alpha > 1$ is the mark-up over the competitive real wage (8) a monopoly union would be able to capture. Notice that when unions maximize the objective function (16), real wages are always set above the reservation wage and vary only in response to changes in the reservation wage so that we have real wage rigidity. When unions simply maximize workers’ utility real wages are instead fully flexible.

2.5. Market clearing

Market clearing conditions imply that output is entirely consumed, and therefore

$$Y_t^w = C_{w,t}, \quad Y_t^u = C_{u,t}$$

(21)

Since the net supply of bonds in equilibrium is zero, equilibrium in the bonds market implies

$$B_t = 0$$

(22)

From Eqs. (10) and (12) we have

$$D_t^w Y_t^w = A_t(N_t^w)^2 \quad \text{and} \quad D_t^u Y_t^u = A_t(N_t^u)^2$$

(23)

where

$$D_t^w = \left[ \int_0^q \frac{1}{q} \left( \frac{P_t^w(j)}{P_t} \right)^{-\theta / \alpha} dj \right]^2 \quad \text{and} \quad D_t^u = \left[ \int_0^1 \frac{1}{1-q} \left( \frac{P_t^u(j)}{P_t} \right)^{-\theta / \alpha} dj \right]^2$$

(24)

are measures of price dispersion. Given the market clearing conditions and given Eq. (5) we have that

$$Y_t = \left( \frac{Y_t^w}{q} \right)^{1-q} \left( \frac{Y_t^u}{1-q} \right)^{1-q}$$

(25)

Since in a neighborhood of a symmetric equilibrium and up to a first-order approximation $D_t^w \simeq 1$, the total amount of goods produced by the economy is a geometric average of the aggregate production of the two sectors.

2.6. The first best level of output

The efficient level of output can be obtained by solving the problem of a benevolent planner that maximizes the intertemporal utility of the representative household, subject to the resource constraint and the production function. This problem is analyzed in the Appendix D where we show that the efficient level of output is given by

$$Y_t^{Eff} = a_t$$

(26)

2.7. The two sector labor market equilibrium

Labor market equilibrium in the Walrasian sector is obtained equating labor demand (15) to labor supply (8), so that

$$-C_t \frac{\phi_{N_t^w}(N_t)}{\phi(N_t)} = MC_t^w \frac{P_t^w}{P_t} \frac{\alpha}{(1 - \tau^w)} N_t^w$$

(27)

From the households’ intertemporal problem (derived in Appendix B) we have $P_t^w C_{w,t} = q P_t C_t$ and, since the market clearing condition implies $C_{w,t} = Y_t^w$,

$$-\frac{\phi_{N_t^w}(N_t^w, N_t^u)}{\phi(N_t^w, N_t^u)} N_t^w = \frac{\alpha q}{(1 - \tau^w)} MC_t^w$$

(28)

Similarly, in the unionized sector, considering the wage schedule (16) and the labor demand (15), equilibrium in the labor market is given by

$$\frac{\zeta}{\zeta - \gamma (1 - \alpha)} \frac{W_t^u}{P_t} = MC_t^u \left( \frac{P_t^u}{P_t} \right)^q \frac{\alpha}{(1 - \tau^u)} N_t^u$$

(29)

Notice that, differently from what happens in the Walrasian sector, Eq. (29) contains the relative price between goods produced in the Walrasian and in the unionized sectors. In the Walrasian labor market the relative price does not affect
equilibrium, since movements in the relative price are corrected by movements in the relative wage. In the unionized sector instead, because of real wage rigidity, a change in the relative price has a significant effect on equilibrium.

Since, from the intertemporal household problem, (Appendix B) we have \( P_{it}^u C_{ui} = (1 - q) C_t P_t \) and given Eq. (6) we have

\[
\frac{\zeta}{\zeta - \gamma(1 - \zeta)} \frac{W_t^f}{P_t} = \frac{\alpha}{\alpha - \tau_w} MC_t^u \left( \frac{1 - q}{q} \right)^q Y_t \frac{N_t^u}{N_t}
\]

(30)

2.8. The flexible price equilibrium output in the Walrasian and in the unionized sectors

The log-linearization of (28) which is shown in Appendix E implies

\[
MC_t^w = \frac{1}{\alpha} \gamma_t^w - \frac{\sigma}{\alpha} \gamma_t - \frac{1}{\alpha} \gamma_t + w_t^w
\]

(31)

Notice that real marginal costs in the Walrasian sector are increasing in the output of the Walrasian sector and decreasing in the aggregate output. Considering that in the flexible price equilibrium we must have \( MC_t^w = 0 \), from the aggregate production function we find

\[
\gamma_t^w = \frac{\sigma}{\alpha} \gamma_t - \frac{1}{\alpha} \gamma_t + w_t^w
\]

(32)

which implies that flexible price equilibrium output in the Walrasian sector is an increasing function of the productivity shock and of the aggregate output. Notice that when \( q = 1 \), (32) can be rewritten as \( \gamma_t^w = \gamma_t \), i.e. the flexible price equilibrium output coincides with the efficient one.

In the unionized sector the log-linearization of (30) implies

\[
MC_t^u = \frac{1}{\alpha} \gamma_t^w - \gamma_t - \frac{1}{\alpha} \gamma_t + w_t^u
\]

(33)

As in the Walrasian sector, real marginal costs are increasing in the output of the unionized sector and decreasing in the aggregate output. When \( MC_t^u = 0 \) then

\[
\gamma_t^u = \frac{\sigma}{\alpha} \gamma_t + \gamma_t - \frac{1}{\alpha} \gamma_t + w_t^u
\]

(34)

which implies that the flexible price equilibrium output in the unionized sector is increasing in the productivity shock and aggregate output.

Given Eqs. (31)–(34), in both sectors real marginal costs can be rewritten in terms of the gap between actual output and flexible price output:

\[
MC_t^w = \frac{1}{\alpha} \gamma_t^w - \gamma_t^w
\]

(35)

2.9. The natural output

We define the natural output of the economy \( y_t^f \) as the weighted sum of flexible price equilibrium output of the Walrasian and unionized sectors (Eqs. (32) and (34)), where the weight is given by the fraction of firms in each sector. Therefore

\[
y_t^f = \frac{\sigma}{\alpha} \frac{q \sigma (\sigma - 1)}{\sigma (1 - \sigma) + 2q} \gamma_t + \frac{\sigma (1 - q) \sigma}{\sigma (1 - \sigma) + 2q} \gamma_t^u
\]

(36)

Given (26), the difference between natural and efficient equilibrium output is

\[
y_t^f - y_t^f = \frac{\sigma (1 - q) \sigma}{\sigma (1 - \sigma) + 2q} \gamma_t + \frac{\sigma (1 - q) \sigma}{\sigma (1 - \sigma) + 2q} \gamma_t^u
\]

(37)

What is important to notice, here, is that, unlike what happens in the Walrasian model, the difference between flexible equilibrium output (natural output) and efficient equilibrium output is not constant, but is a function of the relevant shocks that hit the economy. In this model therefore, as in Blanchard and Gali (2007) stabilizing the output gap—the difference between actual and natural output—is not equivalent to stabilizing the welfare relevant output gap—the gap between actual and efficient output. In other words, what Blanchard and Gali (2007) call “the divine coincidence” does not hold, since any policy that brings the economy to its natural level is not necessarily an optimal policy.

Defining by \( Y' = \sigma (1 - q) / \sigma (1 - \sigma) + 2q \), the response of the welfare relevant output gap to the relevant shocks (notice that the response to a technology shock is identical, but with the opposite sign, to the response to a reservation wage shock), we immediately observe that

\[
\frac{dY}{dq} < 0
\]

(38)
As the number of Walrasian firms increases, the difference between natural output and efficient output decreases, i.e. the natural output tends to the efficient output. The reason is quite intuitive: the smaller is the population of unionized firms, the smaller is the importance of real wage rigidity in the economy and both the technology and reservation wage shocks become less and less relevant.

2.10. The reduced dynamic system

We assume that firms choose \( P_t^f(j) \) in a staggered price setting à la Calvo (1983). The solution of the firm’s problem in the case of a production function with decreasing returns to scale is given by

\[
\pi_t^h = \beta E_t \pi_{t+1}^h + \lambda_a \frac{1}{2} (\psi_t - y_t^h)
\]

(39)

where \( \lambda_a = ((1 - \psi)(1 - \beta)\psi)/(\alpha + \theta(1 - \alpha)) \) and \( \psi \) is the probability with which firms reset prices. Since aggregate inflation is \( \pi_t = q\pi_t^h + (1 - q)\pi_t^w \), and considering the welfare relevant output gap, given by the difference between actual and efficient output \( y_t = y_t^h - y_t^e \), the Phillips curve for the aggregate equation can be written as

\[
\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1}{2} \chi_t + \lambda_a \gamma_t y_t + \lambda_a \gamma_t^w y_t^w
\]

(40)

where we have considered that in log-terms Eq. (17) implies that \( \omega^s = \epsilon^r_w \) and where \( Y = \sigma(1 - q)/(\sigma(1 - \alpha) + aq) \). From Eq. (40) is quite clear that, for a central bank, achieving \( y_t = 0 \) does not imply obtaining \( \pi_t = 0 \).

In order to obtain the IS curve we start by log-linearizing the Euler equation (7) around the steady state. Considering the optimal subsidy setting, which implies \( \phi_t(N)\phi(N) = -\alpha \), the resource constraint (21), the aggregate production function and the definition of the welfare relevant output gap, we obtain

\[
x_t = E_x x_{t+1} - (\hat{r}_t - E_t(\pi_{t+1} - \hat{r}_t^e))
\]

(41)

The interest rate is defined as \( \hat{r}_t = r_t - q \), where \( r_t = \ln R_t \) and \( q = -\ln \beta \) is the steady state interest rate; \( \hat{r}_t^e \) is the interest rate supporting the efficient equilibrium and is given by

\[
\hat{r}_t^e = \sigma E_t[\Delta a_{t+1}] = -\sigma(1 - \rho_a)\bar{a}_t
\]

(42)

The flexible price (natural or Wicksellian) rate of interest is instead defined as

\[
r_t^f = \hat{r}_t^f - \hat{r}_t^e = (1 - \rho_a)\bar{a}_t - (1 - \rho_w)\epsilon^r_w
\]

Suppose that the economy is hit at the same time by a one standard deviation technology shock and by a one standard deviation reservation wage shock. If the persistence of the former is greater (lower) than the persistence of the latter, then, the natural output of the former is greater (lower) than the efficient one. When the shocks have the same persistence, the natural and the efficient rate of interest coincide. Note that, the difference between the natural and the efficient rate of interest is decreasing in the number of Walrasian firms and, in particular, for \( q = 1 \), the endogenous trade-off cancels out and the natural rate of interest is equal to the efficient interest rate.

2.10.1. The endogenous trade-off between inflation and output gap

Let us now consider the nature of the trade-off between inflation and output gap. As we can immediately see from Eq. (40), in contrast to the standard New Keynesian model, a policy that stabilizes inflation does not succeed in stabilizing output at the same time. We can therefore state:

**Result 1.** In a two sector labor market economy, because of the presence of unions, the “divine coincidence” does not hold, i.e., stabilizing inflation is not equivalent to stabilizing the welfare relevant output gap. A negative (positive) productivity shock has a positive (negative) effect on inflation, while a cost-push shock has an effect on inflation of the same size but with the opposite sign.

In addition, given (38), we immediately observe that the response of inflation to technology and exogenous wage shocks decreases as the fraction of Walrasian firms in the market increases. Therefore, we also state that:

**Result 2.** The response of inflation to a negative productivity shock and to a positive reservation wage shock decreases as the number \( q \) of Walrasian firms increases.

In order to give an intuition of the result, suppose, for example, that a positive productivity shock hits the economy. Efficient output (i.e. the first best level of output) is \( y_t = a_t \), and therefore an increase in productivity will increase efficient output by the same amount. As we can see from Eq. (36) natural output (i.e. the level of output that would prevail in a flexible price equilibrium with a dual labor market) increases more than the efficient one. If the labor market were completely Walrasian, real marginal costs would decrease in response to a positive productivity shock, the labor demand

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18 The equation of the natural interest rate is obtained combining (42), (37) and the IS (of the flexible price equilibrium) as in Woodford (2003).
would shift outwards and real wages would increase. On the contrary in our dual labor market aggregate real wage are sluggish. This means that after a positive technology shock real wages in the unionized sector do not increase, and therefore aggregate real wages, increase less (and firms markup decrease more) than in an economy with a purely Walrasian labor market. Consequently, firms react to a positive technology shock by producing more output than in a Walrasian labor market. As it is evident from Eq. (40), if the central bank stabilizes output around the efficient level, inflation will be completely vulnerable to productivity shocks; in other words, the output gap is no longer a sufficient statistics for the effect of real activity on inflation. As shown by Eq. (37), the larger the union coverage, the larger is the fraction of firms that do not adjust real wages following a productivity shock and therefore the larger is the difference between efficient and natural output. The greater is the size of the unionized labor market, therefore, the greater is the trade-off between inflation and output gap stabilization.

Another interesting aspect of this model is that we are able to express the Phillips curve in its more traditional form, i.e. in terms of unemployment. Let \( U_t = 1 - N_t \) be the rate of unemployment. From the log-linearization of the aggregate production function and from Eqs. (26), (37) and (40) we obtain

\[
n_t = \beta E_t \pi_{t+1} - \frac{\lambda_a}{\eta} u_t - \frac{\sigma(1-q)}{\sigma(1-x) + 2q} \pi_t + \frac{\sigma(1-q)}{\sigma(1-x) + 2q} v_t^w
\]

where \( \eta = N/(1-N) \). The relationship between unemployment and the output gap allows us to consider, indifferently, the output gap and the unemployment rate as policy objectives for the central bank.

3. Monetary policy

In Appendix F we show that also for the non-separable preferences assumed in our framework, consumers’ utility can be approximated up to the second order by the quadratic equation:

\[
W_t = E_t \sum_{t=0}^{\infty} \beta^t \hat{U}_{t+k} = -\frac{U_t}{2} E_t \sum_{t=0}^{\infty} \beta^t \left[ \pi_{t+k}^2 + \frac{\lambda_a}{\eta} \pi_{t+k}^2 \right] + O(\|x\|^3)
\]

where \( \hat{U}_{t+k} = U_{t+k} - \bar{U}_{t+k} \) is the deviation of consumers’ utility from the level achievable in the efficient equilibrium. If the central bank cannot credibly commit in advance to a future policy action or to a sequence of future policy actions, then the optimal monetary policy is discretionary, in the sense that the policy makers choose in each period the value to assign to the policy instrument, that here we assume to be the short-term nominal interest rate \( \bar{r}_t \). In order to do so, the central bank maximizes the welfare-based loss function (44), subject to the economy’s Phillips curve (40) and to the IS curve, (41), taking all expectations as given. The first-order conditions imply

\[
x_t = -\frac{\theta \sigma}{\alpha} \pi_t
\]

Substituting into (40), iterating forward, and considering the law of motion of (13) and (17), we obtain

\[
x_t = -\frac{\theta \sigma}{\alpha(\Omega - \beta \rho_a)} \pi_t - \frac{\theta \sigma}{\Omega - \beta \rho_w} v_t^w
\]

where \( \Omega = 1 + \omega(\theta \sigma / \alpha)^2 \). Notice that we can express current inflation as a function of the relevant shocks \( \pi_t \) and \( v_t^w \). A positive productivity shock requires a decrease in inflation and a positive cost-push shock requires an increase in inflation. Given Eqs. (40) and (46) we can write the expression of the output gap as a function of the exogenous shocks,

\[
x_t = \frac{\theta \sigma y_{a}}{\alpha(\Omega - \beta \rho_a)} \pi_t - \frac{\theta \sigma y_{a}}{\Omega - \beta \rho_w} v_t^w
\]

The optimal level of inflation can be implemented by the central bank by setting the nominal interest rate. The interest rate rule can be obtained by substituting (45), (46) into the IS curve, in which case we obtain

\[
\bar{r}_t' = \left[ 1 + \left( \frac{1 - \rho_a}{\rho_a} \frac{\theta \sigma y_{a}}{\Omega - \beta \rho_a} \right) \pi_t + \sigma(1 - \rho_a) a_t \right]
\]

\[
+ \left[ 1 + \left( \frac{1 - \rho_w}{\rho_w} \frac{\theta \sigma y_{a}}{\Omega - \beta \rho_w} \right) v_t^w
\]

\[
\right]

We can therefore state:

**Result 3.** Under discretion an optimal monetary policy requires a decrease in the nominal interest rate following a positive productivity shock and an increase in the nominal interest rate following a positive reservation wage shock. The response of the nominal interest rate to both shocks decreases as the fraction of Walrasian firms \( q \) increases.

Notice that, unlike what happens in the standard New Keynesian model, monetary policy should not be neutral in response to a productivity shock, but rather procyclical. The interest rate must decrease more than proportionally following a positive technology shock and must increase more than proportionally following a reservation wage shock. A similar result, but with a different model for the labor market is obtained by Faia (2008) in a model with search and matching
frictions. However, while in that paper optimal monetary policy must be procyclical only when worker’s bargaining power is higher than the share of unemployed people in the matching technology, in our paper the nominal interest rate must always decrease in response to a positive productivity shock.

Let us now turn to the instrument rule that implements the optimal monetary policy. To make things simple we assume that the economy is hit by a shock at the time. Then, when the economy is hit by only a technology shock, i.e. $\epsilon_t^y = 0$, from Eq. (46) and the equation of the IS curve we obtain

$$\hat{r}_t = \hat{r}_t^* + \left[ 1 + \frac{1 - \rho_w}{\rho_w} \right] \frac{\theta \sigma}{\alpha} E_t \pi_{t+1}$$

(49)

Analogously, when the economy is hit by a single reservation shock and $a_t = 0$, then the instrument rule becomes

$$\hat{r}_t = \hat{r}_t^* + \left[ 1 + \frac{1 - \rho_w}{\rho_w} \right] \frac{\theta \sigma}{\alpha} E_t \pi_{t+1}$$

(50)

Also in our economy, therefore, the Taylor principle always applies, i.e.

**Result 4.** Optimal monetary policy under discretion requires a more than proportional increase in the nominal interest rate following an increase in the expected rate of inflation.

In order to better understand the properties of the optimal monetary policy it is worth studying the path of optimal inflation and output gap volatility for different values of the union coverage parameter. Eqs. (46) and (47) can be easily rewritten in terms of standard deviations. Since by assumption both shocks are iid and therefore $\sigma_{aw} = 0$, we can express the volatility of inflation and the volatility of the output gap as a function of the volatility of the technology and reservation wage shocks. In particular we have

$$\sigma_v = \left( \frac{\Gamma^w_a}{\Omega - \beta \rho_a} \right) \sigma_a + \left( \frac{\Gamma^w_d}{\Omega - \beta \rho_w} \right) \sigma_w$$

(51)

and

$$\sigma_x = \left( \frac{\theta \sigma \Gamma^w_a}{\alpha (\Omega - \beta \rho_a)} \right) \sigma_a + \left( \frac{\theta \sigma \Gamma^w_d}{\alpha (\Omega - \beta \rho_w)} \right) \sigma_w$$

(52)

Notice that, as $q \rightarrow 1$ then $\gamma \equiv \sigma(1 - q)/(\sigma(1 - \tau) + \sigma q) = 0$ and therefore $\sigma_x = \sigma_v = 0$. When instead $q \rightarrow 0$, then $\gamma = \sigma/(\sigma(1 - \tau) + \tau)$ and both $\sigma_x$ and $\sigma_v$ reach their maximum possible values.

Given (38) we can easily see that, at the optimum, the volatility of inflation and the volatility of the output gap increases as the number of unionized firm $(1 - q)$ increases. This means that economies characterized by a stronger presence of unions should be allowed to be more volatile than economies with more competitive labor markets. The reason is that, because of real wage rigidity, output reacts more to shocks and therefore tends to be more volatile. Consequently, also the cost of stabilizing inflation, for the central bank, increases and it is optimal to allow a larger volatility of inflation and output.

### 4. Calibration

Since we are interested in how union coverage influences the conduct of monetary policy, we calibrate the model under different values of the union coverage $1 - q$. We start by simulating the IRFs to a one standard deviation productivity shock under the interest rate rule estimated by Smets and Wouters (2003) for the Euro area. The purpose of this exercise is to obtain a better understanding of the role of real wage rigidity in the transmission of monetary policy and to qualitatively evaluate the IRFs of our model economy relative to the IRFs of the European business cycle. A similar exercise can be found also in Zanetti (2007) who, however, considers separately the behavior of unionized economies and those characterized by Walrasian labor markets. Using our model with a dual labor market, instead, we are also able to compare economies with different values of the union coverage. Differently from Zanetti’s model, ours does not allow for human and physical capital accumulation and therefore we can easily study optimal monetary policy. For this reason, in a second step, we calibrate the model under the optimal monetary policy rule and different values of the union coverage. As in Zanetti (2007), the variables of the model are calibrated using data from the Euro area.

The model is calibrated on quarterly frequencies. For the parameters describing preferences, we set the elasticity of intertemporal substitution at $\sigma = 2$. The output elasticity of labor, $\alpha = 0.72$, is based on the estimate of Christoffel et al. (2006). The discount factor $\beta$, the Calvo parameter $\phi$, and the elasticity of substitution among intermediate goods $\theta$, are set at values commonly found in the literature. In particular we set $\beta = 0.99$, $\phi = 0.75$, which implies an average price duration of one year, and finally $\theta = 6$, which is consistent with a 10% markup in the steady state. The persistence of the technology shock $\rho_a$ is set as in Zanetti (2007) i.e. $\rho_a = 0.8476$. As discussed in Zanetti (2007) $N = 0.61$.

We first consider the IRFs under the following Taylor-type rule:

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1 - \phi_r) [\phi_x \pi_{t-1} + \phi_x \pi_{t-1}]$$

(53)
As in Smets and Wouters (2003), the degree of interest rate smoothing is set at $\phi_i = 0.9$, the response of the nominal interest rate to inflation is set at $\phi_p = 1.658$ and the response to output at $\phi_q = 0.148$. We consider three different degrees of union coverage, and in particular we set: $q = 0$ (an economy where the labor market is fully unionized), $q = 0.15$, which is a good approximation of union coverage in most European countries ($1 - q = 0.85$). Finally, we repeat our simulations with an economy in which the percentage of firms belonging to a unionized labor market is low, i.e. $q = 0.85$, as it is in the US.

In Fig. 1 we plot the IRFs of output, output gap, employment, inflation, real wages and nominal interest rate to a one unit standard deviation positive technology shock under the Taylor rule (53). The IRFs relative to the three different values of union coverage are indicated as follows: a union coverage equal to 1, i.e. $q = 0$ (solid line), $q = 0.15$ (dotted line) and $q = 0.85$ (dashed line). In all cases inflation decreases on impact, the nominal interest rate goes down, employment, real wages and output gap decline, while output increases. In particular, note that under the first assumption, $q = 0$, which implies full unionization, our simple model behaves, from a qualitative point of view, like the one proposed by Zanetti (2007). This suggests that adding physical and human capital accumulation does not qualitatively change the model dynamics. When union coverage is equal to 85%, the economy is still volatile and persistent. On the contrary, when most of the labor market is competitive, i.e. $q = 0.85$, the response of the main economic variables to productivity shocks (except for the real wage) is smaller and shocks are less persistent. Moreover, it is worth noticing from Fig. 1 that after a positive technology shock output rises on impact while employment declines. The fact that employment experiences a large decline ($2007$). This suggests that adding physical and human capital accumulation does not qualitatively change the model

...
Differently from the previous case, under the optimal policy the comovement between productivity shocks and the labor input is not negative, as in the economy where the central bank follows a Taylor rule. This positive comovement can be explained by the fact that, under the optimal rule, monetary policy is much more procyclical than under the Taylor rule (53). This allows output to increase more than the increase in productivity and therefore firms, already in the first period, increase labor demand and employment.

5. Conclusions

In this paper we consider a DSGE-NK model where labor is indivisible and there are two types of labor markets that coexist: a Walrasian one and a unionized one where wages are the result of the bargaining between firms and monopoly unions. We found that, with respect to the standard DSGE-NK framework, we are able to account for the existence of significant trade-offs between stabilizing inflation and stabilizing unemployment, in response to technology and exogenous wage shocks. Because of real wage rigidity, which is induced by the presence of unions, an optimizing central bank must respond to positive technology shocks by increasing the interest rate and, similarly, must respond with an interest rate increase to exogenous increases in unions’ reservation wage.

The effect of these shocks on inflation and the necessary interest rate movements set by an optimizing central bank depend on the size of the Walrasian sector relative to the unionized sector. If a large part of wages are set in a competitive market, technology and cost-push shocks will have little effect on inflation and will induce small interest rate movements, while an economy where large part of wages are set in unionized markets will experience larger inflation and interest rate movements. If we consider, however, an optimal instrument rule where the central bank reacts to expected inflation, the response of the nominal interest rate to an increase in expected inflation is not influenced by the dualistic structure of the labor market.

Even though, for the sake of simplicity, we concentrate on a rigid dualistic structure of the labor market and we abstract from other market imperfections like search and matching and hiring–firing costs we are able to single out, with this model, some of the challenges provided to monetary policy by different institutional settings in the labor market. The model, in particular, captures an important difference between Anglo-Saxon economies and continental Europe providing, therefore, a useful benchmark to evaluate and compare the monetary policies enacted by the Fed, the Bank of England and the ECB.

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Appendix A. Derivation of the representative agent’s utility function

Recalling that \( q \) is the probability of belonging to the Walrasian sector and \( 1 - q \) is the probability of belonging to the unionized sector, before the lotteries are drawn and before learning in what sector they will happen to work, given (1) and
(2) the expected intratemporal utility function of a household is

\[
\frac{1}{1 - \sigma} \left\{ \begin{align*}
q^w_t [C_{0t}^w t_0]^{1-\sigma} + q(1 - N_t^w) [C_{1t}^w t_1]^{1-\sigma} \\
+(1 - q)N_t^w [C_{0t}^w t_0]^{1-\sigma} + (1 - q)(1 - N_t^w) [C_{1t}^w t_1]^{1-\sigma}
\end{align*} \right\}
\]  

(A.1)

Perfect risk sharing implies

\[
\begin{align*}
(C_{0t}^w)^{-\sigma} v(0) &= (C_{1t}^w)^{-\sigma} v(1) \\
(C_u)^{-\sigma} v(0) &= (C_u^w)^{-\sigma} v(0) \\
(C_{0t}^u)^{-\sigma} v(0) &= (C_{1t}^u)^{-\sigma} v(1)
\end{align*}
\]

(A.2)

which imply

\[
C_{0t}^u = C_{0t}^w = C_{0t}^w \quad \text{and} \quad C_{1t}^u = C_{1t}^w = C_{1t}
\]

The average consumption level can be then rewritten as

\[
C_t = [qN_t^w + (1 - q)N_t^u] C_{0t} + [q(1 - N_t^w) + (1 - q)(1 - N_t^u)] C_{1t}
\]

(A.3)

the first two equations of the perfect risk sharing conditions can also be rewritten in a more compact way as

\[
C_{0t}^\sigma v_0^{1-\sigma} = C_{1t}^\sigma v_1^{1-\sigma}
\]

(A.4)

Solving (A.4) for \(C_{1t}\) we get

\[
C_{0t} = C_{1t} \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma}
\]

(A.5)

Substituting (A.5) in (A.3) and solving for \(C_{0t}\)

\[
C_t = [qN_t^w + (1 - q)N_t^u] C_{1t} \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^w) + (1 - q)(1 - N_t^u)] C_{1t}
\]

(A.6)

Solving (A.6) for \(C_{1t}\)

\[
C_{1t} = \frac{C_t}{[qN_t^w + (1 - q)N_t^u] \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^w) + (1 - q)(1 - N_t^u)]}
\]

(A.7)

substituting (A.7) in (A.3)

\[
C_{0t} = \frac{C_t}{[qN_t^w + (1 - q)N_t^u] \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^w) + (1 - q)(1 - N_t^u)]} \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma}
\]

(A.8)

substituting (A.7) and (A.8) in (A.1)

\[
C_t^{1-\sigma} \left\{ [qN_t^w + (1 - q)N_t^u] \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^w) + (1 - q)(1 - N_t^u)] \right\}^{1-\sigma}
\]

\[
\times \left\{ [qN_t^w + (1 - q)N_t^u] \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^w) + (1 - q)(1 - N_t^u)] \right\}^\sigma
\]

(A.9)

Eq. (A.9) can be rewritten as

\[
C_t^{1-\sigma} \left\{ [qN_t^w + (1 - q)N_t^u] \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^w) + (1 - q)(1 - N_t^u)] \right\}^\sigma
\]

(A.10)

defining

\[
\phi(N_t) = [qN_t^w + (1 - q)N_t^u] \left( \frac{b_t}{b_1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^w) + (1 - q)(1 - N_t^u)]^{1-\sigma}
\]

we can write the agents’ intertemporal utility function as Eq. (6) in the text.
Appendix B. Intertemporal allocation

Given the representative agent utility function (num) in the text, the equation of the aggregate consumption \( C_t = (C_{ut}/q)(C_{wt}/(1 - q))^{1 - q} \) and the budget constraint, which can be rewritten as follows:

\[
P_{ut}C_{ut} + P_{wt}C_{wt} + E_tD_{t+1}B_{t+1} \leq W_{ut}N_{ut} + W_{wt}N_{wt} + B_t + II_t - T_t
\]

(B.1)

The first-order conditions with respect to \( C_{ut} \) and \( C_{wt} \), \( B_t \) and \( B_{t+1} \) imply

\[
qC_t^{-\sigma} \left( \frac{C_{wt}}{q} \right)^{q-1} \left( \frac{C_{ut}}{1 - q} \right)^{1-q} \phi(N_t)^{1-\sigma} = \lambda_t P_t^w
\]

\[
qC_t^{-\sigma} \frac{C_t}{C_{wt}} \phi(N_t)^{1-\sigma} = \lambda_t P_t^w
\]

(B.2)

and

\[
(1 - q)C_t^{-\sigma} \frac{C_t}{C_{ut}} \phi(N_t)^{1-\sigma} = \lambda_t P_{ut}
\]

(B.3)

Moreover we have

\[
\dot{\lambda}_t = E_t \left( \beta \dot{\lambda}_{t+1} R_t \right)
\]

(B.4)

Taking a geometric average of (A.9) and (B.3) with weights \( q \) and \( (1 - q) \) we have

\[
\frac{C_t}{P_t^{1-\sigma}} \phi(N_t)^{1-\sigma} = \lambda_t
\]

(B.5)

and substituting in (A.9) and (B.3) we obtain

\[
P_{wt}C_{wt} = qP_t C_t
\]

(B.6)

and

\[
P_{wt}C_{wt} = (1 - q)P_t C_t
\]

(B.7)

Summing (B.7) and (B.6) it is easy to verify that

\[
P_t C_t = P_{ut} C_{ut} + P_{wt} C_{wt}
\]

(B.8)

Finally, combining (B.2), (B.3) and (B.4) we find the consumption Euler equation in the text.

Appendix C. Derivation of the CPI

For a given the equation for final goods \( Y^h(h \in w, u) \) let \( P^w \) be the price of the goods produced in the Walrasian sector that solves

\[
\min \int_0^q P^w(j)Y^w(j)\,dj
\]

subject to

\[
Y^w = \left( \frac{1}{q} \right)^{1/0} \int_0^q Y^w(j)^{0-1/0} \,dj
\]

From the first-order condition we obtain

\[
\int_0^q Y^w(j)^{0-1/0} \,dj = \lambda^w(j)^{-1} Y^w(j)^{-1/0}
\]

(C.1)

Given the budget constraint, this implies,

\[
\left( \frac{1}{q} \right)^{1/0} \lambda^w(j)^{-1} Y^w(j)^{-1/0} = 1
\]

(C.2)

which, in turn, implies

\[
Y^w(j) = \left( \frac{1}{q} \right)^{1/0} \lambda^w(j)^{-1} Y^w(j)^{-1/0}
\]

(C.3)
We can now write
\[ P^w Y^w = P^w = \int_0^q P^w(j) Y^w(j) \, dj = \int_0^q p^w(j) \left( \frac{1}{q} \right)^{\lambda^0} p^w(j)^{\lambda^0} \, dj \] (C.4)

which implies
\[ \lambda = (P^w)^{1/\theta} \left[ \int_0^q \left( \frac{1}{q} \right)^{\lambda^0} p^w(j)^{1-\theta} \, dj \right]^{-1/\theta} \] (C.5)

Notice now that \( C^w = 1 \) implies
\[ \left( \frac{1}{q} \right)^{1/\theta} \int_0^q Y^w(j)^{(1-\lambda)/\theta} \, dj = \int_0^q \left( \frac{1}{q} \right)^{1/\theta} \lambda^{1-\theta} p^w(j)^{1-\theta} \, dj = 1 \] (C.6)

Combining these last two equations we obtain
\[ P^w = \left[ \frac{1}{q} \int_0^q p^w(j)^{1-\theta} \, dj \right]^{1/(1-\theta)} \] (C.7)

Analogously, \( P^u \) is the price of the goods produced in the unionized sector that solves

\[ \min_{C^u} \int_q^1 p^u(j) Y^u(j) \, dj \]

subject to
\[ Y^u = \left[ \left( \frac{1}{1-q} \right)^{1/\theta} \int_q^1 Y^u(j)^{(1-\lambda)/\theta} \, dj \right]^{\theta/(\theta-1)} = 1 \]

and obtaining
\[ P^u = \left[ \frac{1}{1-q} \int_q^1 p^u(j)^{1-\theta} \, dj \right]^{1/(1-\theta)} \] (C.8)

The consumption-based price index solves the problem of minimizing \( q Y_{w,t} + (1-q) Y \) subject to
\[ Y_t = \frac{1}{q^q(1-q)^{1-q}} (Y_{w,t})^q (Y_{u,t})^{1-q} = 1 \]

hence, substituting the optimal demands (B.6) and (B.7) (together with the two market clearing conditions which imply that \( C_{w,t} = Y^w_t \) and that \( C_{u,t} = Y^u_t \) in the previous equation we obtain
\[ Y_t = \frac{1}{q^q(1-q)^{1-q}} \left( q \frac{P^w_t}{P^u_t} Y_t \right)^q \left( (1-q) \frac{P^w_t}{P^u_t} Y_t \right)^{(1-q)} = 1 \]

and simplifying we have
\[ P_t = (P^w_t)^q (P^u_t)^{1-q} \]

**Appendix D. Derivation of the efficient level of output**

We consider a social planner which maximizes the representative household utility subject to the economy resource constraint and production function as follows:

\[ \max_{N_t} U(C_t, N_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} \phi(N_t)^{1-\sigma} \]

s.t. \( C_t = Y_t \)

\[ Y_t = A_t N_t^\sigma \]

Substituting the constraint into the utility function the problem is

\[ \max_N \frac{1}{1-\sigma} (A_t N_t^\sigma)^{1-\sigma} \phi(N_t)^{1-\sigma} \] (D.1)

the first-order condition requires

\[ (A_t N_t^\sigma)^{-\sigma} Y_t \phi(N_t)^{1-\sigma} = - (A_t N_t^\sigma)^{-\sigma} \phi(N_t)^{-\sigma} \phi_N(N_t) \] (D.2)
simplifying
\[ Y_t \frac{\phi_N(N_t)}{\phi(N)} = -\alpha Y_t \frac{N_t}{N} \] (D.3)

Multiplying both sides of equation for \( N_t/Y_t \) we find
\[ \frac{\phi_N(N_t)}{\phi(N)} N_t = -\alpha \] (D.4)

In order to find an equation for the efficient output we first log-linearizing the previous equation around the steady state as follows:
\[ \left[ \phi_N(N) + \phi_{NN}(N)N N_t \right] N_t (1 + n_t) = -\alpha \left( \phi_N(N) + \phi_{NN}(N) N t \right) \] (D.5)
which can be rewritten as
\[ \phi_N(N) N + \phi_N(N) N N_t + \phi_{NN}(N) N^2 N_t = -\alpha \left( \phi_N(N) + \phi_{NN}(N) N t \right) \] (D.6)

considering the steady state equation \( \phi_N(N) N_t = -\alpha \phi(N) \) and collecting terms in \( n_t \) we obtain
\[ \left( 1 + \frac{\phi_N(N) N_t}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\alpha \phi(N)} \left( \frac{\phi_N(N) N_t}{\phi(N)} \right)^{-1} \right) n_t = 0 \] (D.7)
given that
\[ 1 + \frac{\phi_N(N) N_t}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\alpha \phi(N)} \left( \frac{\phi_N(N) N_t}{\phi(N)} \right)^{-1} \neq 0 \]
we require,
\[ n_t = 0 \] (D.8)
and then from the aggregate production function we obtain Eq. (26) in the text.

**Appendix E. Derivation of the flexible price equilibrium output in the Walrasian sector**

Consider the equation of labor market equilibrium of the Walrasian economy:
\[ \phi_{Nw}(N_t)N_t^{w} = -\frac{\alpha q}{(1-\tau)} MC_t^{w} \phi(N_t) \] (E.1)

and consider that
\[ \phi_{Nw}(N_t) = q \phi_N(N_t) \] (E.2)

At the steady state we have
\[ \phi_N(N) N = -\frac{\alpha}{(1-\tau)} MC^{w} \phi(N) \] (E.3)

Since the utility of leisure, \( \phi(N_t) \), is given by
\[ \phi(N_t) = \left\{ q N_t^{w} + (1 - q) N_t^{w} \left( \frac{v^0}{v^1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^{w}) + (1 - q)(1 - N_t^{w})] \right\}^{\sigma/(1-\sigma)} \] (E.4)
we have the following derivatives:
\[ \phi_{Nw}(N_t^{w}, N_t^{w}) = \frac{\sigma}{1-\sigma} \left\{ q N_t^{w} + (1 - q) N_t^{w} \left( \frac{v^0}{v^1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^{w}) + (1 - q)(1 - N_t^{w})] \right\}^{\sigma/(1-\sigma)-1} \times q \left( \frac{v^0}{v^1} \right)^{(1-\sigma)/\sigma} - 1 = q \phi(N) \] (E.5)
\[ \phi_{Nw}(N_t^{w}, N_t^{w}) = \frac{\sigma}{1-\sigma} \left\{ q N_t^{w} + (1 - q) N_t^{w} \left( \frac{v^0}{v^1} \right)^{(1-\sigma)/\sigma} + [q(1 - N_t^{w}) + (1 - q)(1 - N_t^{w})] \right\}^{\sigma/(1-\sigma)-1} \times (1 - q) \left( \frac{v^0}{v^1} \right)^{(1-\sigma)/\sigma} - 1 = (1 - q) \phi(N) \] (E.6)
since the optimal subsidy is set such that in steady state

The last two derivatives at the steady state gives

The optimal subsidy is set in such a way that

which the equation of real marginal costs in the text.

**Appendix F. The welfare-based loss function**

A second-order Taylor expansion of the period utility around the efficient equilibrium yields

where the generic $\tilde{X}_t = \ln(X_t / X)$ denotes log-deviations from the flexible price equilibrium $X_t$. Moreover, we denote as $\tilde{x}_t = \ln(X_t / X)$ the log-deviation from the efficient steady state.
Considering the flexible prices economy resource constraint,

\[ U_t = \bar{U}_t + \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t + \frac{1}{2} \left( \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t^2 + \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t^2 \right) + \bar{U}_{\bar{N}, \bar{N}} \bar{N}_t + \frac{1}{2} \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t \bar{N}_t \bar{N}_t + O(\|z\|^3) \]

Collecting terms yields

\[ U_t = \bar{U}_t + \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t \left[ \hat{Y}_t + \frac{1}{2} \left( \frac{\bar{U}_{\bar{Y}, \bar{Y}}}{\bar{U}_{\bar{Y}, \bar{Y}}} \right) \bar{Y}_t^2 \right] + \frac{1}{2} \left( \bar{U}_{\bar{N}, \bar{N}} \bar{N}_t^2 + \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t \bar{N}_t \right) + O(\|z\|^3) \]  

(F.2)

Considering that, \( \bar{U}_{\bar{Y}, \bar{Y}} \bar{N}_t \bar{N}_t = (1 - \sigma) \phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t / \phi(\bar{N}_t) = -(1 - \sigma) \alpha \), we have

\[ U_t = \bar{U}_t + \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t \left[ \hat{Y}_t - \frac{\sigma}{2} \bar{Y}_t^2 + \frac{1}{2} \left( \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t^2}{\phi(\bar{N}_t)} - \alpha \right) \right] + O(\|z\|^3) \]

(F.4)

It can be shown that \( \phi_{\bar{N}, \bar{N}}(\bar{N}_t) / \phi(\bar{N}_t) = ((2 \sigma - 1) / \sigma)(\phi_{\bar{N}, \bar{N}}(\bar{N}_t) / \phi(\bar{N}_t))^2 \), hence

\[ U_t = \bar{U}_t + \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t \left[ \hat{Y}_t - \frac{\sigma}{2} \bar{Y}_t^2 + \frac{1}{2} \left( \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t^2}{\phi(\bar{N}_t)} - \alpha \right) \right] + O(\|z\|^3) \]

(F.5)

We now take a first-order expansion of the term \( \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t \) around the steady state:

\[ \bar{U}_{\bar{Y}, \bar{Y}} \bar{Y}_t = U_Y \left( 1 + (1 - \sigma) \hat{y}_t + (1 - \sigma) \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t}{\phi(\bar{N}_t)} \right) + O(\|z\|^2) \]

\[ = U_Y \left( 1 + (1 - \sigma) \hat{y}_t - (1 - \sigma) \alpha \bar{n}_t \right) + O(\|z\|^2) \]

(F.7)

\[ \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t)}{\phi(\bar{N}_t)} \bar{N}_t = \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t)}{\phi(\bar{N}_t)} \bar{N} + \Gamma_n \bar{n}_t + O(\|z\|^2) \]

(F.8)

where

\[ \Gamma_n = \left( \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t}{\phi(\bar{N}_t)} \right) = \left( \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t^2}{\phi(\bar{N}_t)} \right) = \frac{A_n \bar{n}_t + O(\|z\|^2)}{} \]

(F.9)

where

\[ A_n = \frac{1}{2} \left( \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t}{\phi(\bar{N}_t)} \right) \left( \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t^2}{\phi(\bar{N}_t)} \right) - \frac{1}{2} \left( \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t)}{\phi(\bar{N}_t)} \right)^3 \]

given that \( n_t = 0 \), and that \( \phi_{\bar{N}, \bar{N}}(\bar{N}_t) / \phi(\bar{N}_t) = -\alpha \), substituting into the Welfare function,

\[ U_t = \bar{U}_t + U_Y \left( 1 + (1 - \sigma) \hat{y}_t \right) \left[ \hat{Y}_t - \frac{\sigma}{2} \bar{Y}_t^2 + \frac{1}{2} \left( \frac{\phi_{\bar{N}, \bar{N}}(\bar{N}_t) \bar{N}_t^2}{\phi(\bar{N}_t)} - \alpha \right) \right] + O(\|z\|^3) \]

(F.10)

Given the aggregate production function and that the log-deviations of the price dispersion index \( d_t = \hat{Y}_t - \alpha \bar{n}_t \) are of second order, and that

\[ \bar{Y}_t^2 = \alpha^2 \bar{n}_t^2, \quad n_t \bar{n}_t = n_t \bar{Y}_t, \quad \bar{Y}_t \bar{n}_t = \bar{Y}_t \bar{Y}_t \]
considering only terms up to the second order we have

$$U_t = \bar{O}_t + \bar{U}_t \left[ \bar{Y}_t - \bar{N}_t - \frac{\sigma}{2} \bar{Y}_t^2 - (1 - \sigma) \bar{Y}_t^2 \right] + O(||x||^3) \tag{F.11}$$

$$\bar{U}_t \equiv U_t - \bar{U}_t = -\bar{U}_t \left\{ d_t + \frac{1}{2} \left( \frac{2\sigma - 1}{\sigma} \right) \bar{Y}_t^2 \right\} + O(||x||^3) \tag{F.12}$$

As proven by Gali and Monacelli (2005), the log-index of the relative-price distortion is of second order and proportional to the variance of prices across firms, which implies that

$$d_t = \ln \left( \int_0^t \frac{P_i(t)}{P_i(t)} - \bar{\varphi}_t \right) = \frac{\theta}{2} \bar{\vartheta}_t \beta + O(||x||^3) \tag{F.13}$$

proof Gali and Monacelli (2005).

As shown in Woodford (2003), this means that

$$\beta = \left( 1 - \psi \right) \left( 1 - \psi \beta \right)^{-1} \tag{F.14}$$

where $\lambda = (1 - \psi)(1 - \psi / \psi) / \psi$.

Finally, denoting the output gap $\bar{Y}_t$, as in the standard way $x_t$, the welfare-based loss-function can be written as

$$W_t = E_t \sum_{k=0}^{\infty} \beta^k \bar{U}_{t+k} = -\bar{U}_t E_t \sum_{k=0}^{\infty} \frac{\theta}{2} \bar{\vartheta}_t \beta_k^2 + \frac{1}{\sigma} \bar{\varphi}_t \beta_k^2 \right\} + O(||x||^3) \tag{F.15}$$

References


