This paper analyzes the optimal behavior of the Central Bank in an economy characterized by balanced growth. We show how trend-growth affects the dynamics of inflation, the preferences of a welfare-maximizing Central Bank and optimal monetary policy. In particular, we show that the optimal monetary policy response to cost-push shocks is not invariant to trend growth, and that countries with lower trend growth have substantially higher incentives to commit to simple rules, both from a welfare and price-stability perspectives.

1. Introduction

Modern dynamic macroeconomic theory, both in the original real business cycle version and in the more recent New Keynesian framework, views business cycles as fluctuations around a constant linear trend, which are produced by short-run, persistent shocks of various nature. The trend is meant to capture the “balanced growth property” of the neoclassical growth model, according to which, when technology is represented by a constant returns production function with labor augmenting technical progress, macroeconomic variables like output, consumption and the stock of physical capital display a similar average rate of growth over sufficiently long periods of time.

Although trend growth is an essential characteristics of dynamic macroeconomics, however, theoretical models usually abstract from it and set it to zero. New Keynesian models, in particular, tend to focus on monetary policy and on the tradeoff between inflation and short-run deviations of output from its potential level, without worrying about the long-run evolution of potential output. Is this an innocuous simplification?

Indeed, the recent debate on monetary policy suggests that the question of how Central Banks should respond to technological progress is an important issue. Until the burst of the “dot.com” bubble and the recent financial crisis, the ability...
of monetary policy to accommodate the impressive growth in productivity that occurred during the period of the New Economy, was praised as a very important achievement of the Greenspan’s Fed. Recently Orphanides (2000, 2001, 2002, 2003) has argued that the “great inflation” of the 1970s can be explained by the failure of the Federal Reserve to understand the productivity slowdown that characterized the US economy in those years. According to this hypothesis, that challenged the traditional interpretation provided by Barro and Gordon (1983) the over-expansionary policy undertaken by the Fed was due to the underestimation of the output gap rather than to the unwillingness of the Fed to fight inflation in order to avoid a recession.3

In light of these considerations, in this paper we study the relationship among trend growth, inflation dynamics and monetary policy within an otherwise standard New Keynesian model, in which productivity follows a trend-stationary process. Are the implications of the standard Dynamic New Keynesian (DNK) model affected by trend growth? Should the Central Bank explicitly consider the rate of productivity growth in formulating monetary policy? Should a Central Bank behave differently in countries (or time periods) characterized by low productivity growth than they should when the productivity growth is very fast?

We find that once trend growth is allowed for, it affects the slope of the Phillips curve and the preferences of a welfare maximizing Central Bank. Moreover, trend growth is also important for the design of optimal monetary policy: when trend growth is high and cost-push shocks are very persistent, optimal monetary policy requires a lower sterilization of these shocks. Interestingly, this effect becomes most important under commitment. Indeed, trend-growth affects the inflation elasticity to expectations by affecting the effective discount factor, and we show that this effect can be quantitatively sizable. This channel makes the equilibrium under commitment – in which monetary policy is able to affect expectations – more sensitive to trend growth than the one under discretion – in which expectations are, instead, taken as given. Specifically, we show that the lower the rate of productivity growth, the greater the improvement in the tradeoff a Central Bank obtains by committing to a simple policy rule, and therefore the stronger the incentive to commit to such simple rule. Moreover, we find that also the gains from commitment, both in terms of inflation stability and welfare, are a decreasing function of trend growth. In the calibration exercise we perform at the end of the paper we show that these effects may be relevant from a quantitative point of view.

The paper is structured as follows. Section 2 discusses the modeling of trend growth in DSGE models. Section 3 describes the theoretical model and the effects of trend growth on the dynamic system. Section 4, then, derives our main results in terms of implied inflation dynamics and optimal monetary policy, providing also a quantitative assessment. Section 5 finally summarizes and concludes.

2. Modeling trend growth in DSGE models

There are two main ways in which the literature usually introduces secular growth in DSGE models.

One way, as in Nelson and Plosser (1982), is by assuming that technology follows a random walk with a constant drift. If we let $\varphi_t$ denote the index of labor-augmenting productivity, the system displays secular growth whenever in the steady-state $\varphi_t$ grows at the constant (gross) rate $\gamma$, greater than one. The random-walk hypothesis, then, implies a general specification of the kind

$$A_t = A_{t-1} + \epsilon_t^{\alpha}$$

Under this formulation the productivity $\varphi_t$ is driven by a difference-stationary process, and the steady-state gross rate of growth is a function of the drift $\delta \geq 0$ and the persistence parameter $\rho_\alpha \in [0,1]$.4 i.e.

$$\gamma = \exp \left\{ \frac{\delta}{1 - \rho_\alpha} \right\}$$

Indeed, many papers in the literature analyzing monetary policy in New Keynesian economies (see Galì (2003) and Galì et al. (2003), among the others) have chosen this first specification. However, by normalizing $\delta$ to zero, they typically disregard secular growth, while still accounting for the unit root in the data. The vast majority of the papers analyzing monetary policy, instead, use an even greater shortcut by assuming a fully stationary model and considering the implied dynamics as the result of a filtering process (like the HP-filter) that removes both trend growth and unit roots.

The number of papers that, instead, explicitly consider trend growth in DNK model is relatively low. In one of the original contributions to the NK literature, Yun (1996) considers the specification above with $\rho_\alpha = 0$, and shows how the shape of the Phillips Curve if affected by trend growth. Analogously, Sbordone (2002) also considers a pure random-walk specification and therefore derives the Phillips Curve as a function of trend growth. However, while these papers do not disregard trend growth altogether in their setups, they still do not focus on the implications that it has for inflation dynamics and monetary policy making. The only paper that, to our knowledge, studies the effects of a change in trend growth on inflation is Bullard and Eusepi (2005). In a model with capital accumulation and adaptive learning, they show that the “great inflation” might have been caused by a misperceptions of trend productivity growth; monetary policy is implemented by a simple Taylor rule reacting to deviations of output growth from the trend rate.

---

2 See for example Woodford (2003).

3 Ireland (1999) provides some evidence in favor of the Barro-Gordon interpretation of the “great inflation”.

4 For $\rho_\alpha = 0$ we get a pure random walk with drift. See Sbordone (2002) and Yun (1996).
The alternative view, that we take in this paper, is to model productivity growth as characterized by a deterministic linear trend.\(^5\) In particular, the trend-stationary model requires the following specification:

\[
\Delta_t \equiv \gamma_t \Delta_t, \\
\ln A_t = (1 - \rho) \ln A + \rho \ln A_{t-1} + \epsilon_t,
\]

in which the trend growth rate \(\gamma\) is a primitive structural parameter and \(A_t\) generates stationary fluctuations of \(\Delta_t\) around such trend.

In our model we focus on the effects that the trend growth rate \(\gamma\) has on inflation dynamics and optimal monetary policy, abstracting from capital accumulation and learning.\(^6\) We simply study what should be the stance of monetary policy, when the specific trend in productivity growth is observed by the Central Bank just like the other structural parameters. In this respect we follow the approach used, for example, by Ascari (2004) in his study of the effects of trend inflation in the DNK model, and verify how standard results are modified when an often-made assumption, simplifying but counterfactual, is lifted.

Although in this paper we do not explicitly consider a broken-trend model and we simply consider economies characterized by different linear deterministic trends, however, we still can compare different economies, or economies in different stages of their growth process, and we can assess how differences in the average growth rate of productivity may affect not only the long-run properties of a dynamic general equilibrium model but also their implications for short-term dynamics and optimal monetary policy.

Indeed, we show that trend growth may have potentially important effects on the dynamic equations describing the evolution of the economy, and on the design of monetary policy. In order to make the analysis of optimal monetary policy non-trivial, we assume, as in Clarida et al. (2002) that the economy is subject to cost-push shocks originating from a variable markup over labor costs; this gives rise to a relevant tradeoff between inflation and output stabilization.

3. The model

In our model economy a continuum of households, indexed \(j \in [0, 1]\), demands a composite consumption bundle, which is produced by a continuum of monopolistically competitive firms, indexed by \(i \in [0, 1]\).

While identical as to their preferences over consumption and leisure, each household supplies a differentiated labor input \(N_t(j)\). A competitive sector of employment agencies then packs all labor types into composite labor services and rent them to firms. Equilibrium in a monopolistically competitive labor market, finally, determines hours worked and the nominal wage for each labor input \(j\).

While no further frictions on the wage-setting mechanism are assumed,\(^7\) we do assume nominal rigidities in the form of a staggered price-setting mechanism à la Calvo (1983).

3.1. Employment agencies

A competitive sector of employment agencies gathers the different labor types from all households and pack them into composite labor services, using the CRS technology

\[
N_t = \left( \int_0^1 N_t(j)^{(\eta_t-1)/\eta_t} dj \right)^{\eta_t/(\eta_t-1)},
\]

in which \(\eta_t\) is the elasticity of substitution between two different varieties of labor. Such composite labor services are then rented to firms at the wage rate \(W_t\). A zero-profit condition for the competitive employment agencies implies the following relation between the nominal wage paid by the firm \(W_t\) and the nominal wage \(W_t(j)\) paid by the employment agencies to type-\(j\) labor supplier

\[
W_t = \left( \int_0^1 W_t(j)^{1-\eta_t} dj \right)^{1/(1-\eta_t)},
\]

and the following demand schedule for each labor type \(j\)

\[
N_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\eta_t} N_t.
\]

Following Clarida et al. (2002), we assume that the degree of market power \(1/\eta_t\) is common across labor suppliers but can be hit by exogenous shocks that make it time-varying. In equilibrium, these translate into cost-push shocks to the Phillips Curve and imply a monetary policy tradeoff between inflation and output stabilization.

---

\(^5\) Following the work of Perron (1989, 1997), a large body of literature has shown that the linear deterministic trend model can reproduce the serial correlation properties of the data just as well as the random-walk model, provided that the possibility of infrequent structural breaks in the trend is allowed for.

\(^6\) We should point out, moreover, that we do not explicitly model broken trends, leaving the problem of the transition from one trend to the other to future research.

\(^7\) As discussed below, this implies flexible wages and thereby common equilibrium nominal wages and hours worked across households.
3.2. Households and the demand-side

Each household has preferences over leisure and consumption, defined by the period-utility function

\[ U_t = \frac{C_t^{1-\sigma}}{1-\sigma} (1 - N_t(j))^{1-\epsilon}, \]

where real consumption \( C_t \) is a CES composite bundle with elasticity of substitution \( \epsilon > 1 \). In order to have a system of preferences consistent with balanced growth, we restrict the parameters value in the utility function to satisfy \( \sigma, \varphi > 1 \). For future reference, note that \( \sigma \) denotes the inverse of the intertemporal elasticity of substitution in consumption and \( \varphi = \frac{N_t}{N} = \frac{\varphi}{\varphi} \) denotes the inverse of the steady-state elasticity of labor supply.

The consumer seeks to maximize the expected discounted stream of lifetime utility flows subject to a sequence of budget constraints of the form:

\[ P_t C_t + E_t\{\mathcal{F}_{t+1}B_{t+1}\} \leq W_t(j)N_t(j) + PR_t + B_t - T_t, \]

where \( \mathcal{F}_{t+1} \) is the stochastic discount factor pricing the one-period contingent claims \( B_{t+1} \), and \( PR_t \) and \( T_t \) are nominal profits from firms and lump-sum taxes, respectively.

In the monopolistic labor market, each household faces the labor demand coming from the employment agencies. Given the absence of any distortion in the labor market besides monopolistic competition, wages are perfectly flexible and the labor market will therefore clear at the same level of wage and labor supply, for all households \( (W_t(j) = W_t, N_t(j) = N_t) \) for all \( j, t \). As a consequence, the labor supply and Euler equation for consumption read:

\[ \frac{1 - \zeta}{1 - \sigma} \frac{C_t}{1 - N_t} (1 + \mu^\nu_t) = \frac{W_t}{P_t}, \]

\[ E_t\{\mathcal{F}_{t+1}C_t^{\sigma}(1 - N_t)^{1-\epsilon}\} = \beta E_t\left\{ \frac{P_t}{P_{t+1}} C_t^{\sigma}(1 - N_{t+1})^{1-\epsilon}\right\}, \]

in which \( \mu^\nu_t \equiv 1/(\eta_t - 1) \) is the net wage mark-up reflecting the workers’ market power.

The nominal gross return \( (1 + r_t) \) on a safe one-period bond paying off one unit of currency in period \( t + 1 \) with probability \( 1 \) (whose current value is therefore \( E_t\{\mathcal{F}_{t+1}\} ) \) is defined by the following non-arbitrage condition:

\[ (1 + r_t)E_t\{\mathcal{F}_{t+1}\} = 1. \]

3.3. Firms and the supply side

Each firm produces a differentiated brand \( i \) of consumption goods, out of the composite labor services provided by the employment agencies, using the linear production function

\[ Y_t(i) = \alpha_i N_t(i) = \gamma^i A_t N_t(i), \]

in which productivity follows the trend-stationary model specified in Eq. (3).

Given wage flexibility, and assuming that a fiscal authority issues an employment subsidy \( \tau \) that offsets the distortion coming from monopolistic competition, equilibrium real marginal costs will be constant across firms and equal to the real wage-per-efficiency-unit:

\[ MC_t = \frac{(1 - \zeta)W_t}{\gamma^i A_t P_t}. \]

Moreover, these firms set their prices following a Calvo-style staggered mechanism: in each period each firm finds out whether or not she can revise its price, facing an exogenous and constant probability \( \theta \) that the price remains unchanged.

This mechanism implies that the aggregate price level is a CES aggregate of revised and unrevised prices, with weights equal to their respective probability:

\[ P_t = \left[ \theta(P_{t-1})^{1-\epsilon} + (1 - \theta)(P^*_t)^{1-\epsilon} \right]^{1/(1-\epsilon)}, \]

where \( P^*_t \) is the newly-set price for those firms who actually get the chance to revise it.

---

8 Therefore, the consumption bundle and aggregate price-index are

\[ C_t = \left( \int_0^1 C_t(i)^{\epsilon/(1-\epsilon)} di \right)^{(1-\epsilon)/\epsilon} \quad P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}. \]

9 See King et al. (1988a,b) for a discussion on the restrictions that preferences must meet to be consistent with balanced growth.

10 This assumption reduces the overall distortions in the system to the nominal rigidities and makes the flexible-price equilibrium also an efficient one. Refer to the Appendix for the derivation of the optimal subsidy \( \tau \). The optimal target for monetary policy in this case can properly be seen as the correction of the remaining distortion.
Every time a firm has this chance, therefore, the new price she sets takes into account the probability that it will have to be charged for \( k \) more periods with probability \( \theta^k \). Each revising firm, then, sets its own price according to the following, implicit rule:

\[
E_t \sum_{k=0}^{\infty} \theta^k E_t \tilde{Y}_{t+k}(P^*_t(i) - (1 + \mu^i)P_{t+k}MC_{t+k}) = 0,
\]

where \( \mu^i \equiv 1/(\epsilon - 1) \) is the steady-state net mark-up reflecting the firms’ market power \( 1/\epsilon \). Under full price-flexibility (\( \theta = 0 \)), each firm revises its own price in each period and sets it as a constant markup over current nominal marginal costs: \( P^*_t(i) = (1 + \mu^i)P_tMC_t \). In this case, all firms \( i \) are symmetric and the price is therefore common across firms: \( P^*_t(i) = P^*_t = P_t \). Thereby, real marginal costs are constant at their steady-state level:

\[
MC_t = (1 + \mu^i)^{-1}.
\]

Integrating Eq. (12) over the continuum of firms yields the aggregate production function

\[
Y_tD_t = \gamma^D A_t N_t,
\]

where \( D_t \) is the index of relative-price dispersion over the continuum of firms:

\[
D_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} di.
\]

The latter, for non-degenerate distributions, has a second-order log-level, and is the term originating the welfare loss from inflation.

Equilibrium in the labor market, finally, yields the equilibrium real marginal costs

\[
MC_t = \frac{1 - \zeta}{1 - \sigma} \frac{(1 + \mu^i)(1 - \tau)}{\gamma^D A_t} \frac{C_t}{1 - N_t}.
\]

### 3.4. The stationary equilibrium

Given trend growth, in our framework variables like consumption and output inherit a deterministic trend from the productivity index, which prevents the system from converging to a steady-state. For a steady-state equilibrium to be definable, therefore, the system needs to be transformed to ensure stationarity. The obvious transformation in this case is to divide the generic trending variable \( X_t \) by the time-trend, and we denote the transformed variable with a “hat”: \( \hat{X}_t \equiv X_t / \gamma^t \).

In terms of the transformed variables, the system is described by:

\[
E_t \{ F_{t+1} \} \hat{C}_t \frac{1}{1 - N_t} = \beta \gamma^{-\sigma} E_t \left\{ \frac{P_t}{P_{t+1}} \hat{C}_t \frac{1}{1 - N_{t+1}} \right\}
\]

\[
E_t \sum_{k=0}^{\infty} \theta^k E_t \tilde{Y}_{t+k}(P^*_t(i) - (1 + \mu^i)P_{t+k}MC_{t+k}) = 0
\]

\[
\hat{Y}_t = \hat{C}_t + \hat{G}_t
\]

\[
\hat{Y}_tD_t = \hat{A}_t\hat{N}_t
\]

\[
MC_t = \frac{1 - \zeta}{1 - \sigma} \frac{(1 + \mu^i)(1 - \tau)}{\hat{A}_t} \frac{\hat{C}_t}{1 - \hat{N}_t},
\]

in which \( \hat{G}_t \) denotes the exogenous stream of real public expenditures.\(^{11}\)

While the model economy grows indefinitely over time, therefore, the stationary system above converges to a zero-inflation steady-state, in which the average rate of growth \( \gamma \) enters the Euler equation for consumption and the pricing equation of consumption goods: given the non-arbitrage condition (11), at the steady-state Eq. (20) implies

\[
\frac{1}{1 + r} = \beta \gamma^{-\sigma}.\]

\(^{11}\) Note that the system of equilibrium conditions (20)–(26) is the same one that would arise from a re-scaled model in which all trending variables in \( t \) are divided by \( \gamma^t \), i.e. a model in which the consumer problem is:

\[
\max_{(c_t, N_t), B_t, t \geq 0} E_t \sum_{t=0}^{\infty} \beta^{t+1} \frac{C_t}{1 - \sigma} (1 - \mu^i)^{1-\epsilon},
\]

\[
\text{s.t. } P_t\hat{C}_t + \gamma E_t \{ F_{t+1} \} \hat{B}_t \leq \hat{W}_t(j)N_t(j) + \hat{R}_t + \hat{B}_t - \hat{T}_t,
\]

where \( \hat{\beta} = \beta \gamma^{1-\epsilon} \).
3.5. The frictionless equilibrium

We define the frictionless equilibrium as the equilibrium in which not only all firms can revise their price at each time \( (\theta = 0) \), but there is also the absence of inefficient shocks to the workers’ markup \((\mu^w = \mu^w)\). While the former feature implies that the distortion due to nominal rigidities is ruled out, the latter, given the existence of the subsidy rate \( \tau \), implies that also the distortions due to monopolistic competition in the labor and goods markets are corrected. Since these are the only distortions that differentiate the DNK model at hand from the RBC counterpart, this particular equilibrium qualifies as a frictionless one.

In this equilibrium, therefore, the following holds:

\[
\frac{1}{1 + \mu^\theta} = \frac{1 - \beta}{1 - \sigma} (1 + \mu^w)(1 - \tau) \frac{\bar{C}_t}{1 - N_t}.
\]  

(27)

Since the optimal setting of the subsidy rate requires\(^{12}\)

\[
(1 + \mu^w)(1 + \mu^\theta)(1 - \tau) = 1,
\]  

(28)

then the frictionless equilibrium defines the “potential” level of the several variables, coinciding also with their efficient level and denoted with an over-bar:

\[
\bar{Y}_t = \frac{1 - \beta}{1 - \sigma} \frac{N_t}{1 - N_t} \bar{C}_t.
\]  

(29)

3.6. The linear model

In order to give the model a linear representation, we compute a first-order Taylor expansion of the several equilibrium conditions around a zero-inflation steady-state. Denoting with lower-case variables the log-deviations from such steady-state equilibrium \( (x_t \equiv \ln X_t - \ln X) \), the linear system reads:

\[
\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{c+1} - \rho) - \frac{(\gamma - 1) \phi}{\sigma} E_t \Delta n_{t+1},
\]  

(30)

\[
p_t^r = \frac{\theta \gamma}{1 + r} E_t p_{t+1}^r + \left( 1 - \frac{\theta \gamma}{1 + r} \right) (mc_t + p_t),
\]  

(31)

\[
p_t = \theta p_{t-1} + (1 - \theta) p_t^i,
\]  

(32)

\[
s_c \hat{y}_t = \hat{c}_t + s_n g_t,
\]  

(33)

\[
\bar{y}_t = a_t + n_t,
\]  

(34)

\[
mc_t = \hat{c}_t + \phi n_t - a_t + u_t,
\]  

(35)

\[
\bar{y}_t = \hat{c}_t + (1 + \phi) n_t,
\]  

(36)

where \( \rho \) is the long-run interest rate \( (\rho \equiv r = \sigma \ln \gamma - \ln \beta) \), \( \phi \) is the steady-state work/leisure ratio \( (\phi \equiv N/(1 - N)) \), \( s_c \) the reciprocal of the steady-state share of consumption \( (s_c \equiv \bar{Y} / \bar{C}) \), and we set \( g_t \equiv \frac{\bar{C}}{\gamma} \hat{g}_t \) and \( u_t \equiv \mu^w - \mu^w \). The shock on public consumption \( g_t \) and the cost-push shock on wage markup \( u_t \) are supposed to evolve following each an autoregressive process:

\[
g_t = \rho g_{t-1} + \epsilon^g_t,
\]  

(37a)

\[
u_t = \rho u_{t-1} + \epsilon^u_t.
\]  

(37b)

Using the resource constraint (33) and the production function (34), both of which of course apply also in the frictionless equilibrium, we get the following formulation for the real marginal costs

\[
mc_t = (s_c + \phi) \bar{y}_t - s_c g_t - (1 + \phi) a_t + u_t,
\]  

(37)

and the potential output:

\[
\bar{y}_t = \frac{1 + \phi}{s_c + \phi} a_t + \frac{s_c}{s_c + \phi} g_t.
\]  

(38)

Therefore, defining the output gap as the deviation of real output from its frictionless level \( x_t \equiv \bar{y}_t - \bar{y}_t \), we can finally write:

\[
mc_t = (s_c + \phi) x_t + u_t.
\]  

(39)

\(^{12}\) Refer to the Appendix.
Note that, although by construction it represents the log-difference between the transformed real and the transformed potential output, our notion of output gap also equals the log-difference between actual real and actual potential output:

$$x_t \equiv \hat{y}_t - \hat{y}_t = \ln \left( \frac{Y_t}{\bar{Y}_t} \right) - \ln \left( \frac{\bar{Y}_t}{\bar{Y}_t} \right) = \ln Y_t - \ln \bar{Y}_t.$$ 

In other words: the output gap in actual levels is a stationary variable.

Using the resource constraint (33) and the production function (34) into the Euler equation for consumption (30) we get the expression for the IS-schedule:13

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{\sigma_c}{\sigma_n} E_t \Delta \hat{c}_{t+1} - \frac{1}{\sigma_n} (r_t - E_t \pi_{t+1} - \rho) + \frac{(\zeta - 1)\phi}{\sigma_n} E_t \Delta \hat{c}_{t+1},$$

where $\pi_t \equiv p_t - p_{t-1}$ is the net inflation rate and we set $\sigma_c \equiv \sigma_\pi$ and $\sigma_c \equiv \sigma_c - (\zeta - 1)\phi$.

Given the definition of potential output (38), we can alternatively express the IS-type schedule in terms of the output gap, rather than the stationarized output:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_n} (r_t - E_t \pi_{t+1} - \pi_t),$$

in which $\pi_t$ is the natural rate of interest, that is the level of the real rate of interest that would prevail in the frictionless equilibrium

$$\pi_t = \rho + \psi_c E_t \Delta \hat{c}_{t+1} - \psi_c E_t \Delta \hat{c}_{t+1},$$

and where the response coefficients $\psi$’s are given by

$$\psi_c \equiv \frac{(\zeta - 1)(\sigma_c - 1)\phi + \sigma_c(1 + \phi)}{\sigma_c + \phi} > 0$$

$$\psi_c \equiv \frac{s_c(\zeta - 1)\phi + \sigma_c\phi}{s_c + \phi} > 0.$$

Substituting Eq. (31) into Eq. (32) and considering Eq. (26) finally gives the dynamics of the inflation rate, described by the following New Keynesian Phillips Curve (NKPC):

$$\pi_t = \beta \gamma^{1-\sigma} E_t \pi_{t+1} + \lambda m c_t,$$

where $\lambda \equiv (1-\rho)(1-\phi)^{1-\sigma}$ measures the gains, in terms of lower inflation, of a unitary reduction in current marginal costs due to either a contraction in demand ($x_t < 0$) or to a favorable cost-push shock ($u_t < 0$).

4. Results

4.1. Inflation dynamics, stabilization tradeoff and welfare

Given the form of the Phillips Curve (45) defined in the previous section, we can now easily prove that the dynamics of inflation is affected by the average growth rate of productivity $\gamma$.

Result 1. Higher (lower) trend growth implies higher (lower) elasticity of current inflation to marginal costs, and lower (higher) elasticity to future inflation expectations.

Higher average growth, indeed, implies a lower marginal rate of intertemporal substitution for consumption, which translates into a lower effective discount rate; this in turn implies that expectations about the future matter less for both consumers and price-setters; as a result, inflation dynamics shows a lower sensitivity with respect to inflation expectations and higher response to current marginal costs (higher $\lambda$).

Given the existence of inefficient shocks hitting the market power of labor suppliers, real marginal costs in this setting are not proportional to the output gap. As a consequence, the NKPC and hence inflation are affected by the cost-push shock $u_t$:

$$\pi_t = \beta \gamma^{1-\sigma} E_t \pi_{t+1} + \kappa x_t + \lambda u_t,$$

where $\kappa \equiv \lambda (s_c + \phi)$.

In the case of a constant wage markup ($\mu^m = \mu^m$ and therefore $u_t = 0$), the NKPC above would imply that complete stabilization of real output at its potential level ($x_t = 0$) would also achieve full price-stability ($\pi_t = 0$). In this case the optimal monetary policy would consist of having the interest rate follow the dynamics implied by Eq. (42).

Note that the adopted assumptions on preferences and the government imply two IS shocks: on government spending ($g$) and on productivity ($a$). We wish to stress that this specific stochastic structure does not play any role in driving our qualitative nor quantitative results, which would go through even if we disregarded, for example, the role of government spending.
Instead, the (exogenously) time-varying wage markup $u_t$ gives rise to a tradeoff between inflation and output-gap stabilization. Fully accommodating the cost-push shock to maintain real output at its potential level would result in a positive inflation rate, proportional to the wage markup. Indeed, as it can be easily shown by solving forward the NKPC (46), an inefficient shock to the wage markup necessarily produces some variation in the inflation rate, the (current and/or expected) output gap, or both:

$$\pi_t = \kappa \sum_{k=0}^{\infty} (\beta_1^{1-\sigma})^k E_t \{ x_{t+k} \} + \frac{\lambda}{1 - \beta_1^{1-\sigma} \rho_u} u_t. \quad (47)$$

Therefore, should monetary policy decide to fully accommodate the cost-push shock and keep the real output at its potential level at all times ($x_t = 0$ for all $t$), the effect would be completely on the inflation rate

$$\pi_t = \chi u_t, \quad (48)$$

where the coefficient

$$\chi = \frac{1}{1 - \beta_1^{1-\sigma} \rho_u} \frac{(1 - \theta)(1 - \theta \beta_1^{1-\sigma})}{\theta(1 - \beta_1^{1-\sigma} \rho_u)}$$

measures the cost of the tradeoff: the higher $\chi$ the more costly, in terms of inflation volatility, the full stabilization of the output gap, for given variance of the shock ($\text{var} \{ \pi_t \} = \chi^2 \text{var} \{ u_t \}$).

As to this point, notice first that, as usual, the tradeoff is the more costly the more persistent is the cost-push (i.e. the higher $\rho_u$): the multiplier falls in a range between $\lambda$ (if the shock is a white noise) and $1 - \beta_1^{1-\sigma}$ ($\lambda$ if the shock is a random walk).

More interestingly for our purposes, however, notice that in our setting the cost of the tradeoff also depends upon trend growth. The sign of the dependence is ambiguous and depends on the persistence of the cost-push shock relative to the degree of price-rigidity $\theta$.

**Result 2.** If the cost-push shock is sufficiently persistent relative to the degree of price rigidity ($\rho_u > \theta$) higher (lower) trend growth is associated with lower (higher) costs – in terms of inflation volatility – of completely stabilizing the output gap.

**Proof.** The effect of trend growth on the cost of the tradeoff is measured by:

$$\frac{\partial \chi}{\partial \gamma} = \frac{(1 - \theta)(1 - \theta \beta_1^{1-\sigma})}{\theta(1 - \beta_1^{1-\sigma} \rho_u)} \frac{(\theta - \rho_u) \propto \theta - \rho_u. \quad (49)}{\theta - \rho_u}$$

Hence, for $\rho_u > \theta$ the coefficient is decreasing in $\gamma$. □

Indeed, while more persistent cost-push shocks amplify the on-impact effects of a given change in the time-discount factor relevant to price-setters ($\beta_1^{1-\sigma}$), a higher price-rigidity implies a stronger but opposite effect on the elasticity to current marginal costs ($\lambda$) of any variation in the effective discount factor. On the one hand, therefore, a given rise in trend growth $\gamma$ through a fall in the effective discount factor, implies a higher on-impact effect of the shock, proportionally to the degree of price-rigidity; on the other hand, however, the lower discount factor associated with higher trend growth implies a lower weight on future conditions and hence a lower effect on current inflation of the expected persistence of cost-push shocks in the future.

The intuition underlying the ambiguity is the following. Higher trend growth implies a lower average discount factor because faster-growing future output makes people more eager to benefit from it also in the present; as to firms, the longer are they expected to keep their price unrevise (i.e. the higher $\theta$) the greater the effects of the reduction in the time-discount factor, and therefore the higher the elasticity of inflation to current cost-push shocks. This channel accounts for the positive effect, proportional to $\theta$. On the other hand, a persistent cost-push shock also affects current inflation by generating expectations of future inflation, the more so the higher its persistency. The higher the expectations of future inflation (the higher $\rho_u$), therefore, the stronger the impact on current inflation of a reduction in the effective time-discount factor, which weighs such expectations. This channel triggers a negative effect of a rise in trend growth, proportional to $\rho_u$. For $\theta < \rho_u$, then, higher trend growth implies better conditions for a Central Banker, because it implies a less costly tradeoff between inflation and output stabilization and, at the end, a smaller concern.

In the presence of a tradeoff, then, the Central Bank faces a non-trivial problem. The derivation of the optimal monetary policy to deal with such tradeoff requires the definition of a system of preferences for the Central Bank, and the solution of a complete constrained optimization problem.

Following Woodford (2003), we derive the objective function of the Central Bank as a second-order approximation of the utility flow of the representative consumer. In the appendix we show that such welfare criterion, in our framework with non-separable preferences, takes the form of a quadratic form of the following kind:

$$W_t = \sum_{k=0}^{\infty} (\beta_1^{1-\sigma})^k \tilde{U}_{t+k} = -\frac{1}{2} \sum_{k=0}^{\infty} (\beta_1^{1-\sigma})^k \left[ \frac{\gamma}{\kappa} p_{t+k} + \frac{\varphi}{\kappa} s_{t+k} \right] + \mathcal{C}(\|a\|^2), \quad (50)$$

where $\tilde{U}_t = \frac{u_t}{u_{t+\gamma}}$ is the deviation of consumers’ utility from the level achievable in the frictionless equilibrium, expressed as a fraction of steady-state output, and the relative weights to inflation and the output gap are linked to structural parameters reflecting preferences and technology (we set $\varsigma \equiv \varsigma_c + \varphi$).
Taking unconditional expectations of the welfare criterion \((50)\), and letting \(\beta - 1\), we can approximate the expected welfare loss resulting from a suboptimal policy by a linear combination of the variances of inflation and the output gap:

\[
\mathcal{L} = \frac{\epsilon}{\lambda} \text{var}(\pi_t) + \xi \text{var}(x_t).
\]  

\((51)\)

Notice that the relative weight on inflation is decreasing in the trend-growth rate:

\[
\frac{\partial (\epsilon/\lambda)}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left[ \frac{\epsilon \theta}{(1 - \theta)(1 - \theta(1 - \sigma))} \right] = -\frac{\epsilon \theta^2 (\sigma - 1) \beta_t^{1-\sigma}}{(1 - \theta)(1 - \theta(1 - \sigma))^2} < 0.
\]  

\((52)\)

We can therefore state:

**Result 3.** Lower (higher) trend growth implies welfare-based preferences of an optimizing Central Bank with a higher concern towards relative inflation (output) stabilization.

The intuition behind this result is straightforward: the staggered price-setting mechanism implies cross-sectional price dispersion across firms, and this originates a misallocation of resources across sectors leading to the welfare-costs of inflation; to a second-order approximation, the discounted stream of such price dispersion indexes is related to the infinite sequence of future, aggregate squared-inflation (see appendix and Woodford, 2003). As a consequence, a lower average growth rate raises the discount factor and thus makes expected price dispersion more sensitive to future inflation volatility.

We can now exploit this welfare metric to properly evaluate the expected costs of a policy action that fully accommodates the cost-push shock to keep real output at its frictionless level. In this case, indeed, considering Eq. \((48)\), the expected loss in welfare, as a fraction of steady-state output, would be

\[
\mathcal{L}_{FA} = \frac{\epsilon \lambda}{(1 - \beta_t^{1-\sigma} \rho_u)^2} \text{var}(u_t).
\]  

\((53)\)

Notice that, as a consequence of Results 2 and 3, trend-growth affects such welfare loss, in a way that again depends on the relative strength of cost-push shock persistence and price stickiness: if cost-push shocks are sufficiently persistent, then higher trend growth implies a lower welfare loss from full output-gap stabilization. Indeed, it is straightforward to show

\[
\frac{\partial \mathcal{L}_{FA}}{\partial \gamma} = \beta_t^{1-\sigma} \frac{\epsilon (1 - \theta)(\sigma - 1)}{20(1 - \beta_t^{1-\sigma} \rho_u)^2} (\theta - 2 \rho_u + \beta_t^{1-\sigma} \rho_u) \times (\theta - 2 \rho_u + \beta_t^{1-\sigma} \rho_u).
\]  

\((54)\)

4.2. Optimal monetary policy under discretion

In this section we characterize the optimal monetary policy under “discretion”, i.e. when the Central Bank cannot credibly commit in advance to a future policy action or a sequence of future policy actions. In this case the policy makers choose in each period what value to assign to their policy instrument, that here we assume to be the short-term nominal interest rate \(r_t\), taking as given expectations about the future.\(^{14}\)

Formally, the optimal discretionary policy is derived as the minimization of the welfare-based loss function \((50)\) subject to the NKPC \((46)\) and the IS-schedule \((41)\). Solving the problem in two steps, in the first step the Central Bank chooses in advance to a future policy or a sequence of future policy actions. In this case the policy makers choose in

\[
\frac{\epsilon}{\lambda} \pi_t^2 + \xi x_t^2,
\]  

\((55)\)

such that

\[
\pi_t^d = \kappa x_t^d + f_t,
\]  

\((56)\)

where \(f_t \equiv \beta_t^{1-\sigma} E_t \pi_{t+1} + \lambda u_t\).

The first-order condition for an optimum requires

\[
\kappa x_t^d = -\epsilon \pi_t^d.
\]  

\((57)\)

Substituting in the NKPC \((46)\) yields

\[
\pi_t^d = \beta_t^{1-\sigma} E_t \pi_{t+1}^d - \epsilon \kappa \pi_t^d + \lambda u_t,
\]  

\((58)\)

or, solving for \(\pi_t^d\)

\[
\pi_t^d = \beta_t^{1-\sigma} \pi_t^d + \lambda \sigma u_t,
\]  

\((59)\)

\(^{14}\) This qualifies as the main feature of a discretionary equilibrium: if the Central Bank cannot commit to any future behavior, then it cannot affect private expectations, which will therefore be taken as exogenous.
where \( \sigma \equiv \frac{1}{\Gamma(\sigma)} \in (0, 1) \). Solving the resulting stochastic difference equation forward finally yields the optimal response of inflation

\[
\pi_t^d = \lambda \sigma \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \Gamma^{1-\sigma} \sigma)^k u_{t-k} = \frac{\lambda}{\epsilon \kappa + (1 - \beta \Gamma^{1-\sigma} \rho_u)} u_t,
\]

(60)

and the output gap

\[
x_t^d = -\frac{\lambda \epsilon}{\epsilon \kappa + (1 - \beta \Gamma^{1-\sigma} \rho_u)} u_t \equiv -\phi_u u_t.
\]

The optimal monetary policy, therefore, requires the Central Bank to split the volatility loss implied by the tradeoff so as to maximize consumers’ welfare.

Notice, moreover, that in our framework the optimal response of the output gap to the cost-push shock is affected by trend growth, in a way that depends on the relative strength of price stickiness and cost-push shock persistence:

\[
\frac{\partial \phi_u}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left[ \frac{(1 - \theta)(1 - \theta \beta \Gamma^{1-\sigma}) \theta^{-1} \epsilon}{(1 - \theta \beta \Gamma^{1-\sigma} \rho_u) + (1 - \theta)(1 - \theta \beta \Gamma^{1-\sigma}) \theta^{-1} (\sigma + \phi) \epsilon} \right] \propto \theta - \rho_u.
\]

(62)

We therefore obtain:

**Result 4.** If the cost-push shock is sufficiently persistent relative to the degree of price rigidity \( (\rho_u > \theta) \), the optimal discretionary monetary policy associated with a lower (higher) trend growth implies a lower (higher) sterilization of the cost-push shock.

The second step of the solution allows to retrieve the optimal feedback response of the interest rate, plugging the optimal responses (60) and (61) into the IS-schedule (41):

\[
r_t = \rho + \psi_s \Delta a_{t+1} - \psi_y \Delta g_{t+1} + \psi_d^d u_t,
\]

(63)

where the response coefficient to the cost-push shock

\[
\psi_d^s = (\rho_u + (1 - \rho_u) \sigma u) \frac{\lambda \sigma}{1 - \beta \Gamma^{1-\sigma} \rho_u}
\]

is also a function of trend growth, increasing or decreasing depending on the difference \( \theta - \rho_u \).

Notice, however, that the above instrument rule is expressed as a direct response to structural shocks. Expressing the interest rate rule as a response to current inflation shows that the response coefficient is independent of trend growth. Indeed, the latter affects the dynamics of the interest rate only through its effects on inflation dynamics: 15

\[
r_t = \pi_t + (\rho_u + (1 - \rho_u) \sigma u) \pi_t^d = \pi_t + \psi_d^d \pi_t^d.
\]

(64)

To evaluate the welfare improvement implied by adopting an optimal monetary policy instead of pursuing full output stability, notice that the implied variances of inflation and the output gap under the discretionary equilibrium are

\[
\text{var}(\pi_t^d) = \left( \frac{\phi_u}{\epsilon} \right)^2 \text{var}(u_t)
\]

(65)

\[
\text{var}(x_t^d) = \phi_u^2 \text{var}(u_t).
\]

(66)

Consequently, the expected welfare loss in this case amounts to

\[
J_D = \frac{1}{2} \left( \frac{\phi_u^2 \text{var}(u_t) + \xi \phi_u^2 \text{var}(u_t)}{\text{var}(u_t)} \right) = \frac{\epsilon \lambda \sigma}{(1 - \beta \Gamma^{1-\sigma} \rho_u)^2} \text{var}(u_t),
\]

(67)

which is always lower than in the case of full accommodation, given \( \sigma \in (0, 1) \).

Notice that, as in the case of fully accommodative policy, the expected welfare loss under discretion is not independent of trend growth, and the type of this relationship depends again upon the persistence of cost-push shocks. To see this, notice that the numerator in (67) can be written as \( \Gamma(\epsilon) \epsilon \gamma(\sigma) \), and is therefore a strictly increasing function of \( \gamma(\sigma) \), for a given \( \theta \in (0, 1) \).

The denominator, on the other hand, is also strictly increasing in \( \gamma(\sigma) \), the more so the higher the persistence \( \rho_u \), and is independent of \( \gamma(\sigma) \) if the cost-push shock is white noise \( (\rho_u = 0) \). Accordingly, very low levels of persistence make the numerator dominate and \( J_D \) an increasing function of trend growth, while the opposite is true for \( \rho_u \rightarrow 1 \).

4.3. The gains from commitment

In this section we study the welfare gains that the Central Bank obtains by committing to a state-contingent rule of the kind \( x_t^d = -\omega u_t \).

15 This can also be seen from Eq. (75).
As in Clarida et al. (1999), pre-commitment to such a rule allows the Central Bank to benefit from a reduced tradeoff between inflation and output-gap stabilization. This results from the fact that a credible commitment affects expectations about the future dynamics of inflation and thus amplifies the stabilization properties of a monetary policy action.

To see this, notice that, under the state-contingent rule above, the NKPC implies that equilibrium inflation evolves according to:

$$\pi^e_t = \frac{\lambda - \kappa \epsilon}{1 - \beta \gamma^{1-\sigma} \rho_u} \pi_t^e,$$

and that the same NKPC can be expressed as

$$\pi^e_t = \frac{\kappa}{1 - \beta \gamma^{1-\sigma} \rho_u} x^e_t + \frac{\lambda}{1 - \beta \gamma^{1-\sigma} \rho_u} \pi_t^e.$$

Accordingly, the gains in terms of reduced inflation of a given contraction in the output gap under (constrained) commitment are higher than the ones obtained in the case of discretion \(\left(\frac{1}{1 - \beta \gamma^{1-\sigma} \rho_u} > \kappa\right)\). This implies an improved tradeoff between inflation and output stabilization, insofar as it allows the Central Bank to achieve a given reduction in inflation with a smaller intervention on the output gap.

Moreover, taking the ratio of the gains in reduced inflation per unit of output loss \(\left(\frac{1}{1 - \beta \gamma^{1-\sigma} \rho_u}\right)\) as the measure of this tradeoff improvement, it is easy to show the following:

**Result 5.** The improvement in the tradeoff implied by the commitment to a rule of the kind \(x^e_t = -\omega u_t\) is the larger (smaller) the lower (higher) the rate of trend growth.

The intuition follows directly from the fact that a lower trend growth implies a higher marginal rate of intertemporal substitution in consumption which in turn implies that expectations about future inflation play a stronger role in determining the dynamics of current inflation. Since the tradeoff improvement is due precisely to the ability of a committed Central Bank to affect expectations, therefore, the higher the elasticity of current inflation to such expectations, the larger the improvement.

The first-order condition for an equilibrium under constrained commitment requires:

$$x^e_t = -\frac{\epsilon}{(1 - \beta \gamma^{1-\sigma} \rho_u)} \pi^e_t.$$

As usual, the improved tradeoff requires a more aggressive response to inflation, as it becomes clear by comparing Eqs. (70) and (57). Moreover, unlike in the case of discretion, the Central Bank’s response to inflation under commitment is not invariant with the rate of trend growth.

**Result 6.** When the Central Bank is committed to a rule of the kind \(x^e_t = -\omega u_t\), the lower (higher) trend growth the stronger (milder) the response to inflation.

This latter result can be seen also in terms of the implied optimal instrument rule, in which the response coefficient to current inflation is decreasing in the trend growth of productivity:

$$r_t = \overline{\pi}_t + \left(\rho_u + (1 - \rho_u) \frac{\sigma_u^\epsilon}{(1 - \beta \gamma^{1-\sigma} \rho_u)}\right) \pi^e_t = \overline{\pi}_t + \psi^\epsilon \pi^e_t.$$

The implied variance of inflation and the output gap under constrained commitment is, therefore:

$$\text{var}(\pi^e_t) = \left(\frac{\lambda(1 - \beta \gamma^{1-\sigma} \rho_u)}{\kappa \epsilon + (1 - \beta \gamma^{1-\sigma} \rho_u)^2}\right)^2 \text{var}(u_t),$$

$$\text{var}(x^e_t) = \left(\frac{\lambda \epsilon}{\kappa \epsilon + (1 - \beta \gamma^{1-\sigma} \rho_u)^2}\right)^2 \text{var}(u_t).$$

We now turn to the evaluation of the gains from commitment. First of all we assess the effects in terms of inflation volatility of adopting a discretionary approach rather than a commitment to a state-contingent rule.

To this aim we analyze the effects of commitment on the ratio between the variance of inflation under discretion and the one under commitment:

$$\frac{\text{var}(\pi^d_t)}{\text{var}(\pi^e_t)} = \left(\frac{\kappa \epsilon + (1 - \beta \gamma^{1-\sigma} \rho_u)^2}{\kappa \epsilon (1 - \beta \gamma^{1-\sigma} \rho_u) + (1 - \beta \gamma^{1-\sigma} \rho_u)^2}\right)^2.$$

Fig. 1 plots the variance ratio against trend growth and the persistence of the cost-push shock. Committing to a rule of the kind \(x^e_t = -\omega u_t\), yields a more stable inflation. Moreover, the stability gains from commitment are the higher the lower is the trend growth in productivity (see Fig. 2).
The commitment to a rule of the kind
\[ x_t = -\omega u_t \]
yields a more stable inflation. Such stability gains from commitment are the higher, the lower trend growth and the more persistent the cost-push shock.

**Result 7.** The commitment to a rule of the kind \( x_t = -\omega u_t \) yields a more stable inflation. Such stability gains from commitment are the higher, the lower trend growth and the more persistent the cost-push shock.
As to welfare, the expected loss in consumers’ utility implied by suboptimal policies, in this case amounts to

\[ \mathcal{L}_C = 0.5 \frac{\kappa^2 \text{var}(\pi_t^c) + \zeta \text{var}(\chi_t^c)}{\kappa + (1 - \beta^\gamma^{1-\sigma} \rho_u)^2 \text{var}(u_t)}. \]  

(75)

As in the previous cases, also under commitment the welfare loss varies with trend growth, in a way that depends on the relative strength of cost-push shock persistence and price rigidity: if cost-push shocks are persistent enough, then higher trend growth is associated with lower welfare losses.

To evaluate the welfare gains implied by commitment, we define the measure of such gains as the relative loss in welfare of moving from the constrained commitment to the discretionary equilibrium:

\[ \mathcal{G} \equiv \frac{\mathcal{L}_D}{\mathcal{L}_C} = \frac{(1 + \kappa \epsilon \varphi)}{\kappa \epsilon + (1 - \beta^\gamma^{1-\sigma} \rho_u)^2 \text{var}(u_t)}. \]  

(76)

First of all notice that if the cost-push shock evolves as a white noise (\( \varphi = 0 \)) and/or prices are fully flexible (\( \theta = 0 \) and therefore \( \kappa \to \infty \)), the welfare losses implied by the discretionary equilibrium are exactly the same arising under commitment: the gains from commitment are nil (\( \mathcal{G} = 1 \)). The reason is that a cost-push shock that does not display persistence introduces a tradeoff which is limited to the period in which the shock hits. In this case, therefore, being able to affect the expectations about the future does not improve the performance of monetary policy, since the effects of the cost-push shock do not persist into the future. The more persistent the cost-push shock, instead, the higher are the welfare gains that an optimizing Central Bank can achieve by committing to a state-contingent rule, since more persistent shocks imply that current dynamics is more affected by expectations about the future, and therefore the ability to affect such expectations generates a greater improvement on policy performance. Fig. 4 shows, moreover, that such gains are the higher the higher the degree of price-rigidity. Again the reason is straightforward: the longer the expected duration of consumer prices, the stronger the role that expectations about future demand conditions play in the price-setting process, the more effective the ability of the policy makers to affect such expectations.

Moreover, Fig. 4 also shows that the gains from commitment are not invariant with respect to trend growth.

**Result 8.** The welfare gains (with respect to a discretionary policy) implied by commitment to a rule of the kind \( \chi_t = -\omega u_t \) are higher the lower trend growth. This effect is greater the stickier consumer prices and/or the more persistent the cost-push shocks.

Indeed, lower trend growth means higher marginal rate of intertemporal substitution and therefore higher sensitivity of real dynamics to expectations about the future. Since commitment allows to affect those expectations, therefore, the gains that it implies are higher precisely when those expectations are more relevant for the dynamics of welfare-relevant variables.

### 4.4. Quantitative evaluation

In this section we provide a quantitative evaluation of the effects that trend growth has on a number of features discussed above.

### Table 1

Trend growth across OECD countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>400 log(\gamma) (50–90) ('90–07)</th>
<th>( \kappa )</th>
<th>( \beta^\gamma^{1-\sigma} )</th>
<th>Country</th>
<th>400 log(\gamma) (50–90) ('90–07)</th>
<th>( \kappa )</th>
<th>( \beta^\gamma^{1-\sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia\textsuperscript{a}</td>
<td>2.41 (2.68) (2.18)</td>
<td>0.1528</td>
<td>0.9667</td>
<td>Japan</td>
<td>6.37 (7.60) (3.50)</td>
<td>0.1677</td>
<td>0.9307</td>
</tr>
<tr>
<td>Canada</td>
<td>3.16 (3.38) (2.65)</td>
<td>0.1557</td>
<td>0.9596</td>
<td>Korea\textsuperscript{c}</td>
<td>8.96 (8.54) (9.08)</td>
<td>0.1770</td>
<td>0.9086</td>
</tr>
<tr>
<td>Denmark</td>
<td>3.67 (4.18) (2.45)</td>
<td>0.1576</td>
<td>0.9550</td>
<td>Spain\textsuperscript{d}</td>
<td>2.54</td>
<td>0.1533</td>
<td>0.9655</td>
</tr>
<tr>
<td>France</td>
<td>4.38 (4.63) (3.79)</td>
<td>0.1604</td>
<td>0.9484</td>
<td>Sweden</td>
<td>4.57 (3.91) (6.14)</td>
<td>0.1611</td>
<td>0.9467</td>
</tr>
<tr>
<td>Germany\textsuperscript{b}</td>
<td>4.58 (5.02) (3.56)</td>
<td>0.1611</td>
<td>0.9466</td>
<td>UK</td>
<td>3.34</td>
<td>0.1560</td>
<td>0.9590</td>
</tr>
<tr>
<td>Italy</td>
<td>4.29 (5.51) (1.46)</td>
<td>0.1600</td>
<td>0.9493</td>
<td>US</td>
<td>3.23 (2.62) (4.91)</td>
<td>0.1564</td>
<td>0.9580</td>
</tr>
<tr>
<td>Mean</td>
<td>4.29 (4.56) (3.75)</td>
<td>0.1599</td>
<td>0.9495</td>
<td>Std Dev</td>
<td>1.82 (1.87) (2.11)</td>
<td>0.0068</td>
<td>0.0162</td>
</tr>
</tbody>
</table>

**Note:** Entries are average net rates of growth, annualized, in percentage points.


\textsuperscript{a} Data start in 1975.

\textsuperscript{b} Data prior to 1991 refer to West Germany.

\textsuperscript{c} Data start in 1985.

\textsuperscript{d} Data start in 1979.
The second and sixth columns of Table 1 report the annual growth rate in productivity for 12 OECD Countries, for the whole sample available from the US Bureau of Labor Statistics, and for a sample split corresponding to the early 1990s, which is widely recognized as a period of productivity boom for the US. The figures display large differences in across different countries: the cross-sectional mean value is around 4.3% and the cross-sectional volatility is almost 2%. The Republic of Korea, for example, grew almost four times faster than Australia, and Japan almost twice as fast as the US. Differences become even more notable in the second part of the sample, with a lower mean (3.75%) and a higher standard deviation (2.11%): in the past 17 years the US grew over three times faster than Italy, and the latter over six times slower than Korea. Also across time periods there are meaningful variations: in the last two decades the US grew almost twice as fast as in the previous period, while Japan's trend growth halved and Italy's is almost four times lower.

In spite of this diversity, the third and seventh columns of Table 1 display a low variability in the implied slope of the Phillips Curve, \(\phi\), which, in turn, implies an analogously low variation in the relative weight on inflation in the welfare criterion: \(\phi_p/c\). This evidence may be viewed as supporting the disregard that existing literature has shown to trend growth, in evaluating the monetary policy implications of DNK models. Nevertheless, below we show that other implications of trend growth for monetary policy evaluation can be quantitatively relevant.

Specifically, we assess the quantitative impact on the theoretical results of the previous sections by comparing the polar cases of the Republic of Korea and Australia, for the whole sample. The left panel of Fig. 3 evaluates the quantitative implications for the implied stabilization tradeoff, and plots the ratio of two indicators for Australia and Korea (in percentage points), as a function of the cost-push shock persistence \(\rho_u\). The blue line considers the coefficient \(\chi\), measuring the cost of the stabilization tradeoff when monetary policy fully accommodates the cost-push shock (see Result 2); the green line considers the improvement in the inflation-output tradeoff when moving from discretion to constrained commitment (see Result 5). The red, vertical line marks the calibrated value for the degree of price stickiness: \(\theta = 0.75\).

The blue line shows that for \(\rho_u = \theta\) the cost of the tradeoff when fully stabilizing the output gap is the same for Korea and Australia (high and low trend growth, respectively). A \(\rho_u\) high enough, however, makes the stabilization tradeoff for Australia more than twice as costly as for Korea, and this difference is entirely attributable to the different trend growth.

In turn, the green line shows that when cost-push shocks are white noise (\(\rho_u = 0\)), the tradeoff improvement achieved by adopting a constrained commitment is the same in the two countries considered, and therefore independent of trend growth, as argued in the previous section. However, as the persistence in the cost-push shock increases, the figure reveals that the improvement in the tradeoff implied by commitment is up to 160% larger in Australia than it is in Korea, suggesting a much stronger incentive for commitment in countries with a low trend growth than in those with a high one.

The right panel in Fig. 3 plots the Australia-to-Korea ratio (in percentage points) of the optimal response coefficient to inflation under constrained commitment, \(\psi_p\) (see the interest-rate rule (71)), as a function of the persistence coefficient \(\rho_u\). The implication of this panel is that, when cost-push shocks are highly persistent, the interest-rate response to inflation of countries with relatively lower trend growth needs to be substantially – up to 80% – more aggressive (see Result 6). This result seems quantitatively rather relevant: everything else equal, an increase in inflation requiring an optimal response of 25 basis points in Korea would instead require a 45 basis points increase in Australia.

16 See also Hansen (2001).

17 In the computations of this section, the two countries are assumed to be structurally symmetric, except for the rate of trend growth. Calibration: \(\beta = 99\), \(\sigma = 5\), \(\xi = 13\), \(s_k = 8^{-1}\), \(\epsilon = 10\), \(\theta = 0.75\). See Chari et al. (2000).
Monetary policy is evaluated through a welfare criterion, derived as a second-order approximation of consumers’ utility. This paper analyzes optimal monetary policy in an economy characterized by balanced growth where the dynamics of labor productivity is described by a trend-stationary process. We show that differences in trend growth have important implications for the short-run dynamics of inflation. A low rate of productivity growth implies a higher marginal rate of substitution between current and future consumption. Accordingly, the lower is productivity growth, the larger are the effects of inflationary expectations on current inflation and the smaller are the effects of current marginal costs.

Given the quantitatively negligible effect that trend growth has on the slope of the Phillips Curve, these results crucially hinge on the “effective” discount factor $\beta_j^{1-\alpha}$ measuring the inflation elasticity to expectations about the future. Unlike the slope $\kappa$, such discount factor is substantially affected by the diversity of trend growth that we find in the data, as shown by the fourth and eighth columns of Table 1, and ranges from 0.9086 for Korea to 0.9667 for the case of Australia. This explains why the major effects of trend growth are related to the equilibrium under commitment, whose difference with the discretionary case relies precisely on the effects on expectations that the former can achieve.

### 5. Summary and conclusions

The gains that a Central Bank obtains from committing to a simple policy rule, both in terms of inflation stability and welfare, turn out to be quite relevant from a quantitative point of view. All this suggests that trend growth is quite important for an optimizing Central Bank: the lower the growth rate of productivity the stronger the incentive to adhere to simple policy rules.
Appendix A

A.1. Optimal subsidy-setting

The employment subsidy \( \tau \) is set so as to maximize steady-state utility of the representative consumer. Since we are dealing with steady-state levels, then, we need to work with stationarized variables, for which such steady-state exists:

\[
\tilde{C}_t \equiv C_t/\gamma, \quad \tilde{Y}_t \equiv Y_t/\gamma, \quad \tilde{G}_t \equiv G_t/\gamma.
\]

Accordingly we have a maximization problem for the stationary transformation of the original system:

\[
\max_N U(\tilde{C}, N) = \frac{\tilde{C}^{1-\sigma}}{1-\sigma} (1-N)^{-\gamma},
\]

subject to the resource \( (\tilde{Y} = \tilde{C} + \tilde{G}) \) and technology \( (\tilde{Y} = AN) \) constraints.

Equivalently:

\[
\max_N AN^{1-\sigma}/(1-\sigma)(1-N)^{1-\gamma}.
\]

The first-order condition requires:

\[
1 - \frac{\gamma}{1-\sigma} \frac{\tilde{C}}{1-N} = A, \quad (A.1)
\]

that is

\[
1 - \frac{\gamma}{1-\sigma} \frac{\tilde{C}}{1-N} = A. \quad (A.2)
\]

As a consequence, the steady-state equilibrium in the labor market implies

\[
1 - \frac{\gamma}{1-\sigma} \frac{\tilde{C}}{1-N} = A = \frac{\tilde{A}}{\tilde{Y}} (1+\mu^\varphi) (1+\mu^\varphi) = 1. \quad (A.3)
\]

A.2. The welfare-based loss function

This appendix derives the welfare-based monetary policy loss function, following Rotemberg and Woodford (1997). Under the assumption that a subsidy to employment exists and is set according to the previous section, we can define the efficient flexible-price (or frictionless) equilibrium as the equilibrium arising in the case of full price-flexibility \( \varphi = 0 \) and no inefficient shocks to the workers' market power \( \mu^\varphi = \mu^\varphi \). Labeling as "potential" the level of each variable in this equilibrium, and denoting it with an over-bar, the frictionless labor supply reads

\[
\tilde{C}_t = \frac{\tilde{A}_t}{\tilde{Y}_t}, \quad (A.4)
\]

and the aggregate production function is

\[
\tilde{Y}_t = \tilde{A}_t \tilde{N}_t. \quad (A.5)
\]

Therefore, in the frictionless equilibrium we have (multiplying both sides of the labor supply by \( \tilde{N}_t/\tilde{Y}_t \) and using Eq. (A.5):

\[
1 - \frac{\gamma}{1-\sigma} \frac{\tilde{N}_t}{\tilde{Y}_t} \frac{\tilde{C}_t}{\tilde{Y}_t} = \frac{\tilde{A}_t}{\tilde{Y}_t} = 1. \quad (A.6)
\]

Accordingly, a non-stochastic steady-state implies:

\[
1 - \frac{\gamma}{1-\sigma} \frac{N}{\tilde{Y}} \frac{\tilde{C}}{\tilde{Y}} = \frac{AN}{\tilde{Y}} = 1. \quad (A.7)
\]

Letting \( \varphi \equiv \frac{N}{\tilde{Y}} \) and \( \gamma \equiv \tilde{Y}/\tilde{C} \), the above relation implies

\[
1 - \frac{\gamma}{1-\sigma} \varphi = 1. \quad (A.8)
\]

First-order Taylor expansion of Eq. (A.6) yields
\[ \frac{1 - \frac{\zeta}{1 - \sigma}}{1 - \frac{\xi}{1 - \sigma}} \mathcal{C}(\hat{t}_t + \hat{c}_t) = \hat{Y}_t (1 - N)\hat{y}_t - \hat{Y}_t N\hat{n}_t + \mathcal{O}(\|a\|^2) \] (A.9)

\[ \frac{1 - \frac{\phi}{1 - \sigma}}{1 - \frac{s_c}{s_c}} (\hat{n}_t + \hat{c}_t) = \hat{Y}_t - \phi\hat{n}_t + \mathcal{O}(\|a\|^2) \] (A.10)

\[ \hat{c}_t = \hat{y}_t - (1 + \phi)\hat{n}_t + \mathcal{O}(\|a\|^2). \] (A.11)

Considering the aggregate resource constraint \( \hat{Y}_t = \hat{C}_t + \hat{C}_c \), which applies also to the frictionless equilibrium and to first-order implies \( \hat{c}_t = s_c\hat{y}_t - s_c g_t \), where \( g_t = \frac{s_c}{1 - \sigma} \), and the log-linear production function under the frictionless equilibrium \( \hat{y}_t = a_t + \hat{n}_t \), we can finally write the equation for potential output as

\[ \hat{y}_t = \frac{1 + \phi}{s_c + \phi} a_t + \frac{s_c}{s_c + \phi} g_t + \mathcal{O}(\|a\|^2). \] (A.12)

Denoting with \( \bar{X} \equiv \ln(\hat{X}_t / \hat{X}) = \ln(\hat{X}_t / \hat{X}) \) the log-deviation of variable \( X \) from the frictionless equilibrium, a second-order Taylor expansion of period utility around the frictionless equilibrium yields

\[ \bar{U}_t = \bar{U}_t - \bar{U}_Y \frac{\bar{Y}_t}{\bar{Y}} \left[ \bar{Y}_t - \frac{1 - \frac{\zeta}{1 - \sigma}}{1 - \frac{\xi}{1 - \sigma}} \bar{C}_t \bar{Y}_t - \frac{1 - \frac{\zeta}{1 - \sigma}}{1 - \frac{\xi}{1 - \sigma}} \frac{\bar{N}_t}{1 - \bar{N}_t} \frac{\bar{N}_t}{1 - \bar{N}_t} \right] + \mathcal{O}(\|a\|^2). \] (A.13)

Now we take first-order Taylor expansion of the frictionless equilibrium:

\[ \hat{C}_t^{\sigma} (1 - N_t)^{1 - \frac{1}{\gamma}} \hat{Y}_t = \hat{C}^{\sigma} (1 - N)^{1 - \frac{1}{\gamma}} \hat{Y}_t \left[ 1 + (1 - \sigma) \hat{y}_t + \sigma \hat{y}_t \frac{\hat{C}_c}{\hat{C}} g_t - (1 - \zeta) N_t \frac{\hat{n}_t}{1 - N_t} \right] + \mathcal{O}(\|a\|^2), \]

where \( \sigma_c = \sigma s_c \):

\[ \frac{\hat{C}_t}{\hat{Y}_t} \frac{\bar{N}_t}{1 - \bar{N}_t} = \frac{\xi}{s_c} \left[ 1 + (1 + \phi)\hat{n}_t + (s_c - 1) \hat{y}_t - s_c g_t \right] + \mathcal{O}(\|a\|^2); \] (A.14)

\[ \frac{\hat{Y}_t}{\hat{C}_t} = s_c \left[ 1 + (s_c - 1) \hat{y}_t + s_c g_t \right] + \mathcal{O}(\|a\|^2); \] (A.15)

\[ \frac{\bar{N}_t}{1 - \bar{N}_t} = \phi \left[ 1 + (1 + \phi)\hat{n}_t \right] + \mathcal{O}(\|a\|^2). \] (A.16)

Substituting into Eq. (A.13), and considering that \( \bar{U}_t \bar{Y}_t = \hat{C}^{\sigma} (1 - N)^{1 - \frac{1}{\gamma}} \hat{Y}_t \), we get:

\[ \bar{U}_t = \left[ 1 + (1 - \sigma) \hat{y}_t + \sigma g_t - (1 - \zeta) \phi \hat{n}_t \right] \left[ \hat{Y}_t - \frac{1 - \zeta}{1 - \sigma} \frac{\xi}{s_c} \left[ 1 + (1 + \phi)\hat{n}_t + (s_c - 1) \hat{y}_t - s_c g_t \right] \frac{\bar{N}_t}{1 - \bar{N}_t} \right] + \mathcal{O}(\|a\|^2). \] (A.17)

Collecting all terms of order higher than 2 in the residual, using Eq. (A.8) and setting \( \sigma_c = \sigma s_c \):

\[ \bar{U}_t = \hat{Y}_t - s_c \hat{n}_t - (s_c - 1) \hat{y}_t \hat{N}_t - (s_c - 1) \hat{y}_t \hat{N}_t + s_c g_t \hat{N}_t - \frac{\sigma_c - 1}{2} \hat{Y}_t^2 - (1 - \zeta) \phi \hat{Y}_t \hat{N}_t - \frac{1 + \zeta \phi}{2} \hat{N}_t^2 + \left[ \left( 1 - \sigma \right) \hat{y}_t + \sigma g_t - (1 - \zeta) \phi \hat{n}_t \right] \left( \hat{Y}_t - \hat{N}_t \right) + \mathcal{O}(\|a\|^2). \] (A.18)
Given the aggregate production function and the fact that the log-deviation of the price dispersion index \( d_t \) is of second-order, the following always holds (up to second-order):

\[
\begin{align*}
\bar{Y}_t - \bar{Y}_t &= d_t \\
\bar{N}_t - \bar{Y}_t &= \bar{N}_t^2 - \bar{Y}_t^2 \\
\bar{y}_t \bar{N}_t &= \bar{y}_t \bar{Y}_t \\
\bar{N}_t \bar{Y}_t &= \bar{Y}_t^2 \\
g_t \bar{N}_t &= g_t \bar{Y}_t.
\end{align*}
\]

Consequently, the term in square brackets in Eq. (A.18) is of order 3, and the rest (using \( \bar{y}_t = a_t + \bar{n}_t \)) can be cast in the form

\[
\bar{U}_t = -\frac{1}{2} \left( \frac{1}{2} \right) (1 + \varphi) \left( \bar{y}_t - a_t \right) + \left( s_\varphi - 1 \right) \bar{y}_t - s_\varphi g_t = \frac{1}{2} (\sigma_\epsilon + (2 - \zeta) \varphi) \bar{Y}_t^2 + c(\|a\|^2).
\]

Consider now the term multiplying \( \bar{Y}_t \). Using some algebra we get:

\[
\begin{align*}
(1 + \varphi)(\bar{y}_t - a_t) + (s_\varphi - 1) \bar{y}_t - s_\varphi g_t &= (s_\varphi + \varphi) \bar{Y}_t - (1 + \varphi) a_t - s_\varphi g_t = c(\|a\|^2),
\end{align*}
\]

where the last line uses the definition of potential output (A.12).

As a consequence, the second term in the first line of Eq. (A.19) is of third order and goes in the residual, so that the final form of the deviation of consumers’ utility from that emerging in the frictionless equilibrium is

\[
\bar{U}_t = -\frac{1}{2} \left( \frac{1}{2} \right) (1 + \varphi) \left( \bar{y}_t - a_t \right) + \frac{1}{2} (\sigma_\epsilon + (2 - \zeta) \varphi) \bar{Y}_t^2 + c(\|a\|^2),
\]

and \( \xi \equiv \sigma_\epsilon + (2 - \zeta) \varphi = \sigma_\epsilon + \varphi \), where the second equality uses Eq. (A.8).

Lemma 2. The (log) index of relative-price dispersion is of second-order and proportional to the cross-sectional variance of prices across firms:

\[
d_t \equiv \ln \left( \int_0^1 \left( \frac{P_t(f)}{P_i} \right)^{-\epsilon} df \right) = \frac{\epsilon}{2} \text{var}_{\epsilon}(p_t(f)) + c(\|a\|^2)
\]

Proof. Gali and Monacelli (2005). \( \square \)

Lemma 3. Letting \( \tilde{\beta} \equiv \beta \lambda^{-1} \), the discounted value of future price dispersion among firms is proportional to the discounted value of squared-inflation:

\[
\sum_{t=0}^{\infty} \tilde{\beta}^t \text{var}_{\tilde{\beta}}(p_t(f)) = \frac{1}{\lambda} \sum_{t=0}^{\infty} \tilde{\beta}^t \pi^2_t,
\]

where \( \lambda \equiv (1 - \theta)(1 - \theta) / \theta \).

Proof. Woodford (2003, Chap. 6, pp. 399–400). \( \square \)

As a consequence, denoting the output gap \( \bar{Y}_t \) in the familiar fashion \( x_t \), the welfare criterion, which evaluates the discounted stream of (current and expected) utility losses due to nominal rigidities and monopolistic competition, expressed as a fraction of steady-state output, is:

\[
\mathcal{W}_t \equiv E_t \sum_{k=0}^{\infty} \tilde{\beta}^k \bar{U}_{t+k} = -\frac{1}{2} E_t \sum_{k=0}^{\infty} \tilde{\beta}^k \left[ \frac{\epsilon}{\lambda} \pi^2_{t+k} + \xi \pi^2_{t+k} \right] + c(\|a\|^2).
\]

References