SCREENING IN THE CREDIT MARKET

The Role of Collateral

Fabrizio MATTESINI*

Università del Molise and LU1SS, 00198 Rome, Italy

In this paper, we study the role of collateral in the market for business loans when the problem of adverse selection arises. Exploiting the monotonic relationship that exists between the riskiness of firms and their collateral-interest rate trade-off, banks try to induce self-selection among firms. Even when restrictive regularity conditions are imposed on the model, we show that there are serious limits to the possibility of using collateral as a screening device. We prove that the possibility of screening firms according to their riskiness crucially depends on the proportion of low- and high-risk firms, and we study the properties of the second-best contracts that prevail in the market.

1. Introduction

While collateral is usually regarded by banks as an indispensable element of loan contracts, only recently has the role of collateral and its uniqueness with respect to other components of credit contracts been investigated. Bester (1985), Stiglitz and Weiss (1985), and Besanko and Thakor (1987) have suggested that collateral could be useful in dealing with the problem of adverse selection in the credit market. Adverse selection arises when lenders cannot discriminate among firms of different riskiness and when a contract that is optimal when offered only to less risky borrowers also attracts more risky borrowers, thus becoming unprofitable to the lenders.

Provided that a monotonic relationship between riskiness and preferences can be established, these authors show that banks will always be able to screen firms according to their riskiness. While in Bester (1985), screening precludes the possibility of credit rationing, in Besanko and Thakor (1987), the fact that equilibrium contracts are separating does not rule out the possibility that for some borrowers, credit is rationed.

In this paper, we argue that there are serious limits to the use of collateral as a screening device. These limits are also found when restrictive regularity

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1 Stiglitz and Weiss (1985) obtain pooling in equilibrium only when the indifference curves of different groups of firms do not respect the single crossing property.
conditions are imposed on the probability distribution of firms' revenues and even if we assume that agents can easily trade off collateral for interest rates. Allowing for the possibility that banks earn zero aggregate profits on lending to different types of firms, we use a concept of equilibrium that is more general than the one used by Bester (1985) and Besanko and Thakor (1987). Our analysis shows that, when information is asymmetric, the equilibrium that prevails may not be separating and the possibility of screening different types of firms depends crucially on the proportion of high- to low-risk firms in the market.

We first analyze a market for business loans when information is symmetric. Since we assume that there are collateral costs, interest rate and collateral are not perfect substitutes and the use of collateral gives rise to a deadweight loss. The contract that is optimal for both borrowers and lenders is the one that minimizes this deadweight loss and is, therefore, the contract containing no collateral.

Under asymmetric information, banks will try to exploit the different propensities of different types of firms to trade off collateral for interest rates, and will design second-best contracts that attract different types of firms. If the proportion of high-risk firms in the market is sufficiently high, it is optimal for the banks to incur the costs implied by the use of a positive level of collateral and to offer a pair of separating contracts. If the proportion of high-risk firms is low, it may be optimal to offer to both low- and high-risk firms the same contract.

We also analyze how the level of collateral required by banks varies with changes in the proportion of high-risk firms. Given the nonconcavities that are present in the model, general comparative statics results cannot be established. Under a more restrictive set of assumptions, however, we prove that the equilibrium level of collateral that low-risk firms are required to furnish is a monotonic function of the relative proportion between high- and low-risk firms.

To characterize the equilibrium contracts that prevail in the market, we define a concept of equilibrium, that we call $J_1$-equilibrium, which differs from the Rothschild and Stiglitz (1976) equilibrium only in that we allow for cross-subsidization between firms. In order to deal with the problem that such an equilibrium might not exist, we also introduce a concept of anticipatory equilibrium, which we call $J_2$-equilibrium, similar to the equilibrium concept proposed by Wilson (1977). In studying the properties of equilibrium contracts we use the important result that both the $J_1$-equilibrium (when it exists) and the $J_2$-equilibrium are a solution to a well-defined nonlinear programming problem.

Collateral costs are the necessary legal documentation, monitoring and/or insurance for the asset used as collateral to maintain its contractual value as well as implicit costs that firms incur by relinquishing the discretionary use of collateral.
Section 2 describes the model, section 3 analyzes the case of symmetric information and in section 4 we study the case of asymmetric information. In section 4.1 the relevant equilibrium concepts are defined, and in section 4.2 we study the properties of the equilibrium contracts.

2. The model

We assume two groups of firms which differ only by their riskiness. Each firm has a choice between borrowing or not an amount $B$ from a bank. This amount is technologically given and identical for every firm. A firm of type $\theta$ borrows in order to finance a production activity, from which it gets revenue $R$ which is distributed according to a p.d.f. $F(R, \theta)$, with a continuous density $f(R, \theta)$ on the support $[0, T]$. The parameter $\theta$ characterizing the probability distribution function is a riskiness parameter. Higher $\theta$ means higher risk for the firm in the sense of first-order stochastic dominance. Indexing high-risk firms by $\kappa$ and low-risk firms by $\ell$, this means that

\begin{enumerate}
\item $\theta_\kappa > \theta_\ell$;
\item for all $R$ such that $T > R > 0$, $F(R, \theta_\kappa) > F(R, \theta_\ell)$;
\item $F(0, \theta_\kappa) = F(0, \theta_\ell) = 0$, $F(T, \theta_\kappa) = F(T, \theta_\ell) = 1$.
\end{enumerate}

We denote by $r$ the interest factor on the loan (1 plus the interest rate), by $C$ the amount of collateral firms are asked to pledge and by $I$ the interest factor paid by banks on deposits. In order to insure that there are always gains to be obtained from undertaking a production activity, we assume that

$$\int_0^T R f(R, \theta_i) \, dR > IB, \quad i = \kappa, \ell. \quad (2)$$

Depending on the realization of $R$ each firm may decide to repay its debt or may default. If $R - rB \geq -C$, the firm will repay the loan, and if $R - rB < -C$, the firm will default. Calling $\hat{R}$ the lowest value above which the firm will repay the loan, we define, for each $r, C$

$$\hat{R} = \hat{R}(r, C) = rB - C. \quad (3)$$

Thus, when the firm does not default, its profits are given by the difference between revenues and costs (in our case, the principal plus the interest on the debt) $R - rB$. When it defaults, the firm loses its collateral $C$ plus the revenues obtained from the one-period investment project. We also assume that furnishing collateral is costly for the firm and that such costs are a proportion $\delta$ of the amount of collateral required. Thus expected profits from a loan at interest $r$ and collateral $C$ to a firm of type $i$ are
(4)

Since costs associated to the use of collateral are paid both when the loan is repaid and when bankruptcy occurs, they do not affect the bankruptcy decision of the firm. Consider a bank's profit from lending to a firm. If the firm does not default, expected profits from lending are the interest on the loan plus the principal minus the amount paid to depositors. If the firm defaults, the bank's profit is the value of the collateral plus the revenues realized minus the amount paid to depositors. Thus, the bank's expected profits from lending to a firm of type \( i \) at interest \( r \) and collateral \( C \) are

\[
\pi_i(r, C) = \int_0^R (R-rB)f(R, \theta)\,dR - \int_0^\delta Cf(R, \theta)\,dR - \delta C. \tag{5}
\]

Let us now assume that both banks and firms are risk-neutral, expected value maximizers. To get better insight into the problem, consider the preference map of the typical lender and borrower. Suppose \( \hat{R}(r, C) > 0 \) (which implies \( F(\hat{R}, \theta) > 0 \)) and, for each firm

\[
\frac{dC}{dr}|_{\pi = h} = -(1-F(\hat{R}, \theta))B/F(\hat{R}, \theta) + \delta < 0,
\]

\[
\frac{d^2C}{dr^2}|_{\pi = h} = (1 + \delta)f(\hat{R}, \theta)B^2/[F(\hat{R}, \theta) + \delta]^2 > 0,
\]

i.e., firms' isoprofit curves are downward sloping and convex to the origin in the \((r, C)\) space where lower isoprofit curves mean higher profits to the firms. If \( rB \leq C \), isoprofit curves have slope \(-B/\delta\). By simple differentiation we can easily prove the following lemma.

**Lemma 1.** (i) For values of \( r \) and \( C \) such that \( T > rB - C > 0 \),

\[
|\frac{dC}{dr}|_{\pi = h} - |\frac{dC}{dr}|_{\pi = h} > 0.
\]

(ii) For values of \( r \) and \( C \) such that \( rB - C \leq 0 \),

\[
|\frac{dC}{dr}|_{\pi = h} - |\frac{dC}{dr}|_{\pi = h} = 0.
\]

Under the assumption of first-order stochastic dominance, low-risk firms' isoprofit curves are steeper than high-risk firms' isoprofit curves, i.e., low-risk firms, when \( r \) increases, can keep their expected profits constant only by decreasing \( C \) by more than high-risk firms. High-risk firms, in fact, always have a higher probability of default, so that the marginal value of collateral
to them is lower than to low-risk firms. As we will see in section 4, this direct 
correlation between the riskiness of a firm and its collateral-interest rate 
trade-off is very important when banks cannot observe firms' revenues and 
face the problem of screening different types of firms according to their 
riskiness.

Consider now the banks' isoprofit curves. Suppose \( \hat{R} = rB - C > 0 \), which 
implies \( F(\hat{R}, \theta) > 0 \). It follows that

\[
\frac{dC}{dr}_{|\mu_k} = - B(1 - F(\hat{R}, \theta))/F(\hat{R}, \theta) < 0,
\]

\[
\frac{d^2C}{dr^2}_{|\mu_k} = f(\hat{R}, \theta)B^2/F(\hat{R}, \theta)^2 > 0.
\]

Again, if \( rB \leq C \), then \( F(\hat{R}, \theta) = 0 \) and banks' isoprofit curves are vertical 
lines. Notice that banks' isoprofit curves are always steeper than firms' 
isoprofit curves. Another very important property of the model is given by 
the following.

**Lemma 2.** \( \rho_\ell(r, C) \geq \rho_\hat{\epsilon}(r, C) \), with \( (\geq) \) if \( \hat{R} > 0 \).

Banks' expected profits from offering a contract to low-risk firms are 
higher than the expected profits from offering the same contract to high-risk 
firms. The possibility of ranking the profits of banks from lending to various 
types of firms is essential to the definition of a suitable concept of 
equilibrium under asymmetric information.

### 3. The case of symmetric information

Assume now that there is a large number of banks in the market, that the 
banking system is competitive and that banks and firms have the same 
information about firms' riskiness. In this case, an equilibrium is given by the 
pair of contracts \((r_\delta, C_\delta), (r_\ell, C_\ell)\) that solves the following programming 
problem for each \( i = \delta, \ell \)

\[
\pi_i(r_i, C_i) = \int_{R_i}^T \int_{R_i}^T (R - r_iB) f(R, \theta_i) dR - \int_{0}^{R_i} f(R, \theta_i) dR - \delta C_i,
\]

subject to

\[
\rho_i(r_i, C_i) = \int_{R_i}^T r_iBf(R, \theta_i) dR + \int_{0}^{R_i} f(R + C_i) dF(R, \theta_i) - IB = 0.
\]

Substituting (9) into (8), the problem reduces to choosing the pair \((r_i, C_i)\) 
that maximizes
Denoting by \((r_i^*, C_i^*)\), \(i=A, \ell\), the pair of contracts that solves problem (8)--(9), we now establish the following.

**Proposition 1.** \(C_\ell^* = C_A^* = 0\); \(\rho_\ell(r_\ell^*, C_\ell^*) > \rho_A(r_A^*, C_A^*)\); and \(r_A^* < r_\ell^* < T/B\).

When all agents are risk neutral, information is symmetric and collateral is costly for firms, no collateral is required in credit contracts. In fact, the costs incurred by firms in furnishing collateral represent a deadweight loss that can be avoided only with a contract that implies no collateral. The equilibrium contracts are represented by points \(w^*_A\) and \(w^*_\ell\) in fig. 1.

### 4. The case of asymmetric information

#### 4.1. Equilibrium concepts

We will now analyze the case in which banks are not able to discriminate among different types of firms and we will study under what conditions a positive level of collateral will be required in order to induce self-selection in
the market. In order to be able to address this problem, we will introduce two suitable concepts of equilibrium. The first one, that we call $J_1$-equilibrium, is similar to the Nash equilibrium concept introduced by Rothschild and Stiglitz (1976). The second one, which we call $J_2$-equilibrium, is similar to the concept of anticipatory equilibrium proposed by Wilson (1977) and, as in Wilson (1977) is introduced to deal with the problem that a Rothschild–Stiglitz equilibrium may not exist. Both concepts of equilibrium differ from the ones proposed by Rothschild and Stiglitz and by Wilson in that they allow banks to lose on a group of firms provided that aggregate profits are nonnegative.\footnote{Definitions of equilibrium where cross-subsidization is allowed for were also used by Miyazaki (1977) and Spence (1977). As we will see later, however, these papers are not sufficiently rigorous in the definition of the equilibrium concepts.}

Let us denote by $w_i = (r_i, C_i)$ a contract offered to firms of type $i = \{A, A\}$. Let $w = (w_A, w_A)$ and define

$$\Omega = \{ w \in \mathbb{R}^2 \times \mathbb{R}^2 : \pi_A(w_A) \geq \pi_A(w_A), \pi_A(w_A) \geq \pi_A(w_A) \} . \quad (11)$$

All the contracts that belong to $\Omega$ are such that there is no incentive, for a firm of a given type, to accept a contract designed for a different type of firm. The set $\Omega$ therefore is the set of all pairs of incentive compatible contracts. We will denote a member of $\Omega$ as an offer. To define a concept of equilibrium, we need to establish how firms behave when a new contract is offered in the market. For this purpose we denote by $\hat{w} \in \Omega$ a pair of contracts initially offered, we denote by $w' \in \Omega$ a counteroffer to the original contract, and we define by $J: \Omega \times \Omega \rightarrow 2^\mathcal{F}$ the set of firms that move to a new contract when the pair of contracts $w'$ is introduced in the market. Different specifications of $J(\hat{w}, w')$ will allow us to define different concepts of equilibrium. Let $Y = \{A, A\}$ be the set of firms in the market and $\mu_i$ the proportion of firms of type $i$. We can now state the following definition.

**Definition 1.** Given $J(\cdot, \cdot)$, a pair of contracts $\hat{w} = (\hat{w}_A, \hat{w}_A) \in \Omega$ is a $J$-equilibrium if:

1. $\sum_{i \in I} \mu_i \pi_i(\hat{w}_i) \geq 0$;
2. $\pi_i(\hat{w}_i) \geq \pi_i(w_i^*)$, for $i = \{A, A\}$;
3. there is no $w' \in \Omega$ such that $\sum_{i \in I} \mu_i \pi_i(w_i') > 0$, where $J = J(\hat{w}, w')$.

Part (i) of Definition 1 requires that, in equilibrium, a pair of contracts produce for the banks nonnegative profits. Notice that with this definition we allow banks to lose by lending to one type of firms, provided that the aggregate profits remain nonnegative.

Part (ii) of Definition 1 is essential to the definition of equilibrium. As it is stated in Proposition 1, a bank earns zero profits if it offers the contract $w_i^*$
to high-risk firms and makes positive profits if it offers $w^*_e$ to low-risk firms. If in equilibrium the profits of firms of each type were lower than ones firms could earn on $w^*_e$, firms’ profits could be increased simply by offering $w^*_e$ to both types of firms, in which case the original contract could not be an equilibrium. More formally, assume $\pi_i(\hat{w}_i) < \pi_i(w^*_e)$. Then, given (10) and the continuity of the $\pi(\cdot)$ and $p(\cdot)$ functions, there exists a pair of contracts $w'$ such that (a) $\pi_i(w') > \pi_i(\hat{w}_i)$ for $i = h, l'$; (b) given the monotonicity of preferences, $\rho_h(w') \geq \rho_h(w^*_e)$ and $\rho_l(w') \geq \rho_l(w^*_e) = 0$. This means that it is always possible to find a contract pair such that each contract is preferred to the equilibrium contract by each firm and earns zero profits for the bank. Therefore, if $\pi_i(\hat{w}_i) < \pi_i(w^*_e)$, $\hat{w}$ cannot be an equilibrium.

Part (iii) of Definition 1 requires that no other contract is expected by banks to earn positive profits. According to this definition of equilibrium, banks not only consider the profitability of existing contracts, but also form some expectations about the effect of new offers on the existing contracts. These expectations are reflected in the function $J(\cdot, \cdot)$, which describes how firms react to the introduction of a new contract.

Let us consider now the $J_1$-equilibrium concept. In order to define the appropriate $J$ function, for any given $\hat{w} \in \Omega$, $w' \in \Omega$ and $i = h, l'$, we define the function $W(\hat{w}, w') : \Omega \times \Omega \to \Omega$ by

(a) $W_i(\hat{w}, w') = w'_i$ if $\pi_i(w'_i) > \pi_i(\hat{w}_i)$,

(b) $W_i(\hat{w}, w') = \hat{w}_i$ otherwise.

Interpret $W_i(\hat{w}, w')$ as the contracts which would be purchased by firm $i$ if $\hat{w}$ were originally offered and $w'$ were added, so that both $w'$ and $\hat{w}$ would be available to borrowers. Notice that we are assuming that firms will abandon the old contract and move to the new one if and only if the new contract is strictly more profitable than the old one.4 Let us now define the function $J_1 : \Omega \times \Omega \to 2^Y$ such that $J_1(\hat{w}, w') = \{i \in Y : W_i(\hat{w}, w') \neq \hat{w}_i\}$.

A pair of contracts $\hat{w} \in \Omega$ is a $J_1$-equilibrium if it satisfies Definition 1 for $J = J_1$.

According to this concept, an equilibrium is defined as a set of contracts such that a bank earns nonnegative profits on all contracts and there is no other contract that, if offered, would give positive profits to the bank.5 In order to explain this concept of equilibrium, assume that banks are offering the pair of contracts $\hat{w}$ which gives them nonnegative profits in the aggregate and that a new offer $w'$ is introduced. If such contracts are strictly more profitable to the firms than the existing contracts, firms will move to $w'$. For the original offer $\hat{w}$ to be an equilibrium it is sufficient that none of the new

4This assumption is not essential and is introduced only to simplify the analysis.

5This concept of equilibrium corresponds to the subgame perfect equilibrium of a game where banks simultaneously offer a pair of contracts after which firms choose their best contract.
contracts introduced, which are preferred by firms, earns nonnegative profits to the banks.

Under this definition of equilibrium, the behavior of banks is extremely myopic. Suppose, for example, that the new offer $w'$ is such that $\pi_d(w') > \pi_d(\hat{w})$ and that $\pi_d(\hat{w}) \leq \pi_d(\hat{w}_d)$. In this case low-risk firms will move to $w'$, and the banks offering the original pair of contracts will be left only with high-risk customers. If $\rho(\hat{w}_d) < 0$, then these banks will lose. A $J_1$-equilibrium, however, requires that banks do not react to the effects of a new offer, but rather, assumes that all existing contracts will remain in the market. It follows that the original pair of contracts may not be an equilibrium. Given the static nature of agents expectations, as in Rothschild and Stiglitz (1976), a $J_1$-equilibrium may not exist.

The $J_2$-equilibrium concept, analogous the one proposed by Wilson (1977), is introduced in order to avoid the problem of the nonexistence of an equilibrium. Denote by $w^*_d$ the contract that maximizes $\pi_d(w_d)$ subject to $\rho(w_d) = 0$. Letting $J_1 = J_1(\hat{w}, w')$, define a function $J_2: \Omega \times \Omega \to 2^\Omega$ such that:

(a) if $\sum_{i \in I_1} \mu_i \rho_i(W_i(\hat{w}, w')) \geq 0$, then $J_2(\hat{w}, w') = J_1(\hat{w}, w')$;
(b) otherwise, $J_2(\hat{w}, w') = \{i \in I: \pi_i(w_i) > \pi_i(w^*_d)\}$.

A pair of contracts $\hat{w} \in \Omega$ is a $J_2$-equilibrium if and only if it satisfies Definition 1 for $J = J_2$.

Definition 1 and the definition of a $J_2$ function, imply together that a set of contracts is a $J_2$-equilibrium if each bank earns nonnegative profits and there is no other set of contracts which earns positive profits, after all the unprofitable contracts have been withdrawn. Part (a) of the definition of a $J_2$ function tells us that, if banks earn nonnegative profits by lending to the firms that remain in the old contract, then $J_1$ and $J_2$ coincide. In part (b) of the definition we take into account the possibility that the existing contracts, made unprofitable by some new offer, are withdrawn and a fall-back contract is introduced. The fall-back contract is such that a bank earns nonnegative profits on each type of firms. In this case $J_2$ is the set of all the firms that make more profits on the new contract than on the fall-back contract offered by the bank. A $J_2$-equilibrium exists also when a $J_1$-equilibrium does not.

In order to explain the $J_2$-equilibrium concept, suppose again the $\hat{w}$ is an original offer and that $w'$ is a counteroffer introduced by some banks. If $w'$ attracts some types of firms but fails to produce nonnegative profits to banks, then $\hat{w}$ will not be withdrawn and will be an equilibrium. Suppose, as in the previous case, that $w'$ is such that $\pi_d(w') > \pi_d(\hat{w})$ and that $\pi_d(\hat{w}) \leq \pi_d(\hat{w}_d)$, so that only the high-risk firms will keep borrowing from the banks offering the original contract. If $\rho(\hat{w}_d) < 0$, then $\hat{w}$ becomes unprofitable. Unlike in the case of a $J_1$-equilibrium, under this concept of equilibrium we assume that if $\hat{w}$ becomes unprofitable, the banks offering $\hat{w}$ will react by withdrawing the existing contracts and will use a fall-back contract, i.e., will
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offer \( w_\#^* \) to both types of firm. If there are no other contracts which are preferred to this fall-back contract by firms and which produce positive profits to banks, then this contract will prevail.

The fact that banks make nonnegative profits when they offer the fall-back contracts to each type of firms is very important. In fact, only when banks introduce a fall-back contract with this characteristic are we able to rule out the possibility that a new offer will make any fall-back contract unprofitable. The definition of a \( J_2 \)-equilibrium implies that banks, before offering any new contract, anticipate the chain of reactions we just described. If they expect that the fall-back offer will prevail in the market, then they will never introduce in the first place an alternative to the original contract and \( \hat{w} \) is a \( J_2 \)-equilibrium. The specification of a fall-back contract was missing in the papers by Miyazaki (1977) and Spence (1977). 6

The \( J_2 \)-equilibrium concept has two important properties that will be very helpful in our analysis. The first one is given by

\textit{Proposition 2.} If a pair of contracts \( \hat{w} \) is a \( J_1 \)-equilibrium, then it is also a \( J_2 \)-equilibrium. 7

Given this proposition, we will be able to use the \( J_2 \)-equilibrium concept, knowing that the results obtained will be identical to the ones obtained under the more standard \( J_1 \)-equilibrium concept, when such an equilibrium exists. Consider now the problem of choosing the pair \((w_\#, w_\hat{\#})\) such that:

\textit{Problem I}

\[
\text{maximize } \pi_A(w_\hat{\#})
\]

subject to

\[
\pi_A(w_\#) \geq \pi_A(w_\#^*), \quad (12)
\]

\[
\pi_A(w_\#) \geq \pi_A(w_\hat{\#}), \quad (13)
\]

6The need to specify a fall-back contract derives from the assumption that banks can offer contracts to more than one type of firms. Suppose again \( \hat{w} = (\hat{w}_A, \hat{w}_\#) \) is the original contract and that some bank introduces a new offer \( w = (w_\#^*, w_\#) \in \Omega \) that attracts only low-risk firms and that earns nonnegative profits for the banks even when it is accepted by both types of firms. If \( \hat{w} \) is withdrawn and no fall-back contract is introduced, high-risk firms will move to \( w \) and there will be a possibility that some lender again offers another contract. By simply withdrawing \( \hat{w} \) without using a fall-back contract, banks may not be able to create an incentive for competitors to refrain from making an additional offer. When banks instead are assumed to offer only one contract as in Wilson (1977), the withdrawal of the original offer is sufficient to render a counteroffer unprofitable.

7A proof of Proposition 2 is given in Mattesini (1989) – see also Appendix.
Another important property of the \( J_2 \)-equilibrium concept is given by

**Proposition 3.** A pair of contracts \((\hat{w}_h, \hat{w}_c)\) is a \( J_2 \)-equilibrium if and only if it is solution to Problem 1.\(^8\)

Notice that the structure of Problem 1 reflects the main properties of the \( J_2 \)-equilibrium concept. Constraint (12) restates property (ii) of Definition 1. Constraints (13) and (14) are incentive compatibility constraints, i.e., they require that, in equilibrium, there is no incentive, for a type of firm, to move to the contract designed for another type of firm. Constraint (15) requires that banks earn zero aggregate profits from lending to both types of firms.

### 4.2. Equilibrium contracts

#### 4.2.1. Preliminary results

Given Proposition 3 above, we are now able to study the properties of the set of contracts that constitute a \( J_2 \)-equilibrium. The strategy we will follow is to reduce Problem 1 to a more manageable form and to then derive the set of contracts that solves such a problem. In order to simplify the study of the Problem 1, we will ignore, for the moment, constraint (14) and we will consider the problem of finding the pair of contracts \( w_h = (r_h, C_h), w_c = (r_c, C_c) \) such that:

\[
\text{Problem 2} \quad \begin{align*}
\text{maximize} & \quad \pi_c(r_c, C_c) \\
\text{subject to} & \quad \pi_h(r_h, C_h) \geq \pi_h(r'_h, C'_h), \quad (12') \\
& \quad \pi_h(r_h, C_h) \geq \pi_h(r_c, C_c), \quad (13') \\
& \quad \mu_h \rho_h(r_h, C_h) + \mu_c \rho_c(r_c, C_c) = 0. \quad (15')
\end{align*}
\]

We will first characterize the level of collateral offered to high-risk firms and we will prove that, in equilibrium, constraint (13) must be binding. We

\(^8\)A proof of Proposition 3 is given by the proofs of Proposition 4 and Corollary 1 in Mattesini (1989).
will then show that the same pairs of contracts that solve Problem 2 are also a solution to Problem 1. Let us now denote by \((\hat{r}_h, \hat{C}_h), (\hat{r}_f, \hat{C}_f)\) the contract that solves Problem 2. We can immediately establish

**Proposition 4.** \(\hat{C}_h = 0\).

Consider fig. 2 and assume that the equilibrium contract offered to high-risk firms is given by the pair \((\hat{r}_h, \hat{C}_h)\) where \(\hat{C}_h > 0\). In this case, since banks' isoprofit curves are steeper than firms' isoprofit curves, going from \(\hat{w}_h\) to \(\hat{w}_h\) along the high-risk firms' isoprofit curve, banks can increase their profits. Thus \(\hat{w}_h\) cannot be an equilibrium and equilibrium requires \(\hat{C}_h = 0\). We then prove the following lemma.

**Lemma 3.** Constraint (13) holds as an equality.

Consider now fig. 3 and assume that the equilibrium pair of contracts is given by the pair \((\hat{w}_h, \hat{w}_f)\). Given that the banks' isoprofit curves are steeper than firms' isoprofit curves, moving from \(\hat{w}_f\) to \(\hat{w}_f\) along the low-risk firms' isoprofit curve, banks can increase their profits keeping low-risk firms' profits constant. Thus, only points on the high-risk firms' isoprofit curve can be an equilibrium. Given Proposition 4 and Lemma 3, Problem 2 can be rewritten
as the problem of choosing the pair of contracts \( (r_{d}, C_{d}), (r_{f}, C_{f}) \) such that:

**Problem 3**

\[
\text{maximize } \pi_{f}(r_{f}, C_{f})
\]

subject to

\[
\pi_{d}(r_{d}, 0) \geq \pi_{d}(r_{d}^{*}, 0) \tag{12a}
\]

\[
\pi_{d}(r_{d}, 0) = \pi_{d}(r_{f}, C_{f}), \tag{13a}
\]

\[
\mu_{d} \rho_{d}(r_{d}, 0) + \mu_{f} \rho_{f}(r_{f}, C_{f}) = 0. \tag{15a}
\]

We can then establish

**Lemma 4.** *The pair of contracts that solves Problem 3 is also a solution to Problem 1.*

Consider fig. 4 and any contract pair like \((\hat{w}_{d}, \hat{w}_{f})\) where \(\hat{w}_{f}\) lies on the same high-risk firms’ isoprofit curve that passes through \(\hat{w}_{d}\). Given the fact
that low-risk firms' isoprofit curves are steeper than high-risk firms' isoprofit curves, we have that any contract such as \( \tilde{w}_f \) always lies below the low-risk firms' isoprofit curve that passes through \( \tilde{w}_h \). This implies that low-risk firms will always prefer \( \tilde{w}_f \) to \( \tilde{w}_h \) and therefore constraint (14) is redundant.

4.2.2. The properties of equilibrium contracts

Usually in models of asymmetric information with only two types of agents, where the slopes of agents' indifference curves can be ranked and where self-selection is possible, \( J_2 \)-pooling equilibria do not exist. In this model, instead, we cannot exclude the possibility that banks offer the same equilibrium contract to both high- and low-risk firms, i.e., that \( \tilde{C} = 0 \), in which case collateral cannot be used as a screening device. Consider for example fig. 5 and assume that both pairs \( (w'_h, w'_f) \) and \( (w'_h, w'_f) \) are potential equilibria. The pair of contracts \( (w'_h, w'_f) \) earns more profits for the bank on low-risk firms than \( (w'_h, w'_f) \), but less profits on high-risk firms so that we cannot infer a priori which pair of contracts will be an equilibrium and suggests that the outcome will depend on the relevant parameters of the model, such as the number of low- and high-risk firms in the market.

Since Problem 3 is not a standard concave programming problem, it is impossible to derive general results concerning the properties of the equilibrium contracts. We are able to identify, however, two important cases that
provide some useful insights on the role of collateral as a screening device. We can first show that there exists a value of the ratio $\mu_e/\mu_h$ such that constraint (12a) is binding and the level of collateral pledged by low-risk firms is positive. In this case screening occurs in the market. More formally, denoting by $(\tilde{r}_l, \tilde{C}_l), (\tilde{r}_h, \tilde{C}_h)$ the pair of contracts that solves Problem 3 when (12a) is satisfied as an equality, we can establish

**Proposition 5.** (i) $C_0 > 0$; (ii) there exists a $\bar{\mu}$ such that $\tilde{C}_l = \bar{C}_l$ and $\tilde{r}_l = \bar{r}_l$ if and only if $\mu_e/\mu_h \leq \bar{\mu}$; (iii) $\bar{C}_l$ represents the upper bound on $\tilde{C}_l$.

The case in which the ratio between low- and high-risk firms is less than $\bar{\mu}$ is illustrated in fig. 6. The fact that constraint (12a) is binding, together with the incentive compatibility constraint (13a), implies that the two contracts must both lie on the isoprofit curve that passes through $w^*_l = (r^*_l, 0)$. Since from Proposition 1 we know that when $w^*_h$ is offered to low-risk firms banks' profits are positive, competition will drive banks to offer a different contract to low-risk firms. In this case equilibrium is given by a separating pair of contracts $(w^*_h, w^*_l)$ where $\tilde{C}_l$ is positive and reaches its upper bound. Notice that in the case analyzed in Proposition 5, banks earn zero profits on each type of firm. Since $\bar{C}_h = C_h^* = 0$ and since constraint (12a) holds as an
equality, then $\hat{r}_d = r_d^*$ and $\rho_d(\hat{r}_d, \hat{C}_d) = \rho_d(\hat{r}_d, \check{C}_d) = 0$. The result obtained in Proposition 5 is identical to the one obtained by Bester (1985). Bester requires in fact that each contract, in order to be an equilibrium, yield zero expected profits to the bank, and he does not allow for the possibility that a bank loses money by lending to a type of firm.

The other important property of the model is expressed by the following

**Proposition 6.** If $\mu_f > (1 + \delta)\mu_h/\delta$, then $\check{C}_f = 0$.

Proposition 6 identifies a critical value of the ratio between low- and high-risk firms above which banks will require no collateral, in which case the equilibrium may be given by a pair like $(\hat{w}_h, \hat{w}_f)$ in fig. 6, where $\check{C}_f$ reaches its lower bound. In order to understand the logic of this result, let us consider a pair of contracts like the pair $(\hat{w}_h, \hat{w}_f)$ in fig. 6. By requiring that low-risk firms furnish a positive level of collateral, banks will be able to move to a higher isoprofit curve and therefore will be able to increase their profits on high-risk firms but, at the same time their profits on low-risk firms will decrease. Given this trade-off, the relative proportion of firms in the market is a crucial element in the banks’ policy. If the number of high-risk firms is low, the deadweight loss incurred with the use of collateral will not

"Notice that Proposition 6 provides only a sufficient condition for $\check{C}_f = 0$ and therefore it is possible that there exist other values of $\mu_f/\mu_h$ which imply pooling equilibria."
be compensated by the benefits deriving from the possibility to screen different types of firms and it is optimal for banks to offer a pooling pair of contracts.

After having established that there exists a value of $\mu_c/\mu_h$ under which $\hat{C}_r = 0$ and a value of $\mu_c/\mu_h$ above which $\hat{C}_r = \hat{C}_r$, it would be desirable to study how the degree of screening changes with changes in $\mu_c/\mu_h$, for $\hat{C}_r \in [0, \hat{C}_r]$. Unfortunately, on the basis of the assumptions we have made on the probability distribution functions, we cannot say much about the solution to Problem 1 when $\mu_c/\mu_h > \hat{\mu}$ and when $\mu_c < (1 + \delta)\mu_h/\delta$. Since Problem 3 is not necessarily a concave programming problem, while its solution may be interior, we cannot rule out the possibility that a solution is found at one of the two extreme values of $C_r$ (i.e., either at $C_r = 0$ or at $C_r = \hat{C}_r$). An interior solution to Problem 1 may be found if we impose some restrictions on the density functions of firms. Let us assume, for example,

$$f(R, \theta_h)/f(R, \theta_c) > 1 - \delta \mu_c/(1 + \delta) \mu_h. \quad (16)$$

We can establish

**Proposition 7.** If (16) is satisfied for every $R$, then there exists an interior solution to Problem 1.

This particular case is extremely interesting. In fact, if (16) is satisfied, we can also prove that the level of collateral that constitutes an interior solution to Problem 1 not only crucially depends on the number of low-risk and high-risk firms in the market, but is a monotonic function of it, i.e., we can prove

**Proposition 8.** If (16) is satisfied for every $R$, then $d\hat{C}_r/d(\mu_c/\mu_h) < 0$ for $(1 + \delta)/\delta < \mu_c/\mu_h < \hat{\mu}$.

For very high values of $\mu_c/\mu_h$, $\hat{C}_r = 0$ and no screening occurs; for lower values of $\mu_c/\mu_h$, instead, if (16) is satisfied, $\hat{C}_r$ increases monotonically as $\mu_c/\mu_h$ decreases up to a point where $\hat{C}_r = \hat{C}_r$. The level of screening in the credit market becomes therefore a monotonic function of the ratio of high-risk firms to low-risk firms.

5. Conclusions

We have studied a model of the credit market where information is...
asymmetric and where furnishing collateral is costly for firms. Differently from other models that analyze the role of collateral when an adverse selection problem arises in the market for business loans [Bester (1985), Besanko and Thakor (1987)], we allow banks, in equilibrium, to lose by lending to one type of firm, provided that they earn nonnegative aggregate profits. The more general equilibrium concept employed in this paper produces a larger number of possible equilibrium outcomes. In particular, we show that the ability of banks to use collateral as a screening device depends crucially on the proportion of high- and low-risk firms in the market. Since firms face collateral costs, by requiring a positive level of collateral that may induce self-selection, banks produce a deadweight loss for the economy. If the number of high-risk firms is low, it may be optimal for banks not to incur in such a loss and to offer to both types of firms the same contract. In this model therefore, unlike other self-selection models where there exists a monotonic relationship between preferences and riskiness, there are serious limits to the use of collateral as a screening device.

Appendix

Proof of Lemma 1. (i) If \( r_B - C > 0 \), then

\[
\frac{dC}{dr}\bigg|_{\sigma_i=k} - \frac{dC}{dr}\bigg|_{\sigma_d=k} = (1+\delta)\frac{F(\bar{R}, \theta_d) - F(\bar{R}, \theta_d)}{[F(\bar{R}, \theta_d) + \delta][F(\bar{R}, \theta_d) + \delta]} < 0.
\]

Since for each firm of type \( i \) \( dC/dr < 0 \), it follows that

\[
\left|\frac{dC}{dr}\right|_{\sigma_i=k} - \left|\frac{dC}{dr}\right|_{\sigma_d=k} > 0.
\]

(ii) If \( r_B - C \leq 0 \), then \( F(R, \theta) = 0 \) and \( \frac{dC}{dr}\bigg|_{\sigma_i=k} = \frac{dC}{dr}\bigg|_{\sigma_d=k} = -1/\delta \).

Q.E.D.

Proof of Lemma 2. Integrating by parts we obtain

\[
\rho_\sigma(r, C) - \rho_\delta(r, C) = rB(1 - F(\bar{R}, \theta_\sigma)) + RF(R, \theta_\sigma)\big|_0^R + CF(\bar{R}, \theta_\delta)
\]

\[
- \int_0^R F(R, \theta_\sigma) \, dR - rB(1 - F(\bar{R}, \theta_\delta)) - RF(R, \theta_\delta)\big|_0^R
\]

\[
- CF(\bar{R}, \theta_\delta) + \int_0^R F(R, \theta_\delta) \, dR
\]
by the definition of first-order stochastic dominance. Notice that if \( R > 0 \), \( \rho_x(R, C) > \rho_\theta(R, C) \). Q.E.D.

**Proof of Proposition 1.** The problem of choosing the pair \((r_i, C_i)\) which maximizes

\[
\int_0^T R \, dF(R, \theta_i) - \delta C_i - IB
\]

is equivalent to minimizing \( \delta C_i \). Equilibrium is found at \( C_i^\ast = 0 \). The interest rate is the \( r_i^\ast \) that solves (9) when \( C_i = C_i^\ast = 0 \). We know, from Lemma 2, that \( \rho_x(r_i^\ast, 0) \geq \rho_\theta(r_i^\ast, 0) \) with \( > \) if \( \hat{R}(r_i^\ast, 0) > 0 \). Notice that \( \hat{R}(r_i^\ast, 0) = r_i^\ast B > 0 \). If \( r_i^\ast \leq 0 \), in fact \( \rho_x(r_i^\ast, C_i^\ast) = r_i^\ast B - IB < 0 \), which implies a contradiction. Therefore \( \rho_x(r_i^\ast, C_i^\ast) > \rho_\theta(r_i^\ast, C_i^\ast) \). Since \( \partial \rho_x / \partial r > 0 \), and \( \rho_x(r_i^\ast, 0) = \rho_\theta(r_i^\ast, 0) = 0 \) it follows that \( r_i^\ast > r_i^\ast \).

Notice that since \( C_i^\ast = 0 \) \((i = R, \ell)\), given eq. (4), we have \( \pi_x(r_i^\ast, C_i^\ast) + \rho_x(r_i^\ast, C_i^\ast) = \int_0^T R \, dF(R, \theta_i) - IB > 0 \). Since \( \rho_x(r_i^\ast, C_i^\ast) = 0 \), it follows that \( \pi_x(r_i^\ast, C_i^\ast) > 0 \), which can be true if and only if \( r_i^\ast < T/B \). Q.E.D.

**Proof of Proposition 2.** Suppose \( \hat{C}_\ell > 0 \) and consider the pair of contracts \((\hat{r}_\ell, \hat{C}_\ell)\) and \((\hat{r}_\ell + \epsilon_1, \hat{C}_\ell - \epsilon_2)\) such that \( \pi_x(\hat{r}_\ell, \hat{C}_\ell) = \pi_x(\hat{r}_\ell + \epsilon_1, \hat{C}_\ell - \epsilon_2) \), where \( \epsilon_1, \epsilon_2 \) are arbitrarily small numbers. This pair of contracts satisfies constraints (12) and (13).

Observe now that

\[
\rho_x(\hat{r}_\ell + \epsilon_1, \hat{C}_\ell - \epsilon_2) + \pi_x(\hat{r}_\ell + \epsilon_1, \hat{C}_\ell - \epsilon_2)
\]

\[
= \int_0^T R \, dF(R, \theta_\ell) - \delta(\hat{C}_\ell - \epsilon_2) - IB > \int_0^T R \, dF(R, \theta_\ell) - \delta \hat{C}_\ell - IB
\]

\[
= \rho_x(\hat{r}_\ell, \hat{C}_\ell) + \pi_x(\hat{r}_\ell, \hat{C}_\ell).
\]

Since by assumption we have that \( \pi_x(\hat{r}_\ell, \hat{C}_\ell) = \pi_x(\hat{r}_\ell + \epsilon_1, \hat{C}_\ell - \epsilon_2) \), then \( \rho_x(\hat{r}_\ell + \epsilon_1, \hat{C}_\ell - \epsilon_2) > \rho_x(\hat{r}_\ell, \hat{C}_\ell) \), which implies that \((\hat{r}_\ell, \hat{C}_\ell)\) is not an equilibrium. Thus \( \hat{C}_\ell = 0 \). Q.E.D.

**Proof of Lemma 3.** Assume that in equilibrium (13) holds as a strict inequality and consider the pairs of contracts \((\hat{r}_\ell, \hat{C}_\ell)\), \((\hat{r}_\ell + \epsilon_1, \hat{C}_\ell - \epsilon_2)\) such
that \( \pi(\hat{r}_e, \hat{C}_e) = \pi(\hat{r}_e + \varepsilon_1, \hat{C}_e - \varepsilon_2) \), where \( \varepsilon_1, \varepsilon_2 \) are arbitrarily small positive numbers.

In this case,

\[
\rho(\hat{r}_e + \varepsilon_1, \hat{C}_e - \varepsilon_2) + \pi(\hat{r}_e + \varepsilon_1, \hat{C}_e - \varepsilon_2)
\]

\[
= \int_0^T R \, dF(R, \theta_e) - \delta(\hat{C}_e - \varepsilon_2) - IB > \int_0^T R \, dF(R, \theta_e) - \delta \hat{C}_e - IB
\]

\[
= \rho(\hat{r}_e, \hat{C}_e) + \pi(\hat{r}_e, \hat{C}_e).
\]

Since \( \pi(\hat{r}_e, \hat{C}_e) = \pi(\hat{r}_e + \varepsilon_1, \hat{C}_e - \varepsilon_2) \), this implies that \( \rho(\hat{r}_e + \varepsilon_1, \hat{C}_e - \varepsilon_2) > \rho(\hat{r}_e, \hat{C}_e) \). It follows that \((\hat{r}_a, \hat{C}_a), (\hat{r}_e, \hat{C}_e)\) cannot be an equilibrium, which implies a contradiction. Thus (13) must hold as an equality. Q.E.D.

**Proof of Lemma 4.** Assume equilibrium implies \( \hat{r}_a = \hat{r}_a \) and \( \hat{C}_a = \hat{C}_e \). In this case, both constraints (13) and (14) in Problem 2 are satisfied as equalities. Consider then the case in which equilibrium contracts are given by \((\hat{r}_a, \hat{C}_a), (\hat{r}_e, \hat{C}_e)\) where \( \hat{r}_a = \hat{r}_a - \varepsilon_1 \) and \( \hat{C}_e = \hat{C}_e + \varepsilon_2 \) and such that \( \pi(\hat{r}_a, \hat{C}_a) = \pi(\hat{r}_e, \hat{C}_e) \). Given Lemma 1 we know that, when \( r \) increases, in order to keep profits constant, high-risk firms must decrease \( C \) by less than low-risk firms. It follows that \( \pi(\hat{r}_a, \hat{C}_a) < \pi(\hat{r}_e, \hat{C}_e) \). This implies that for any pair of contracts such that (13) is satisfied as an equality, constraint (14) is always satisfied. Given Lemma 3, it follows that constraint (14) is redundant. Q.E.D.

**Proof of Proposition 5.** (i) Consider the pair of contracts \((\hat{r}_e, \hat{C}_e), (\hat{r}_a, 0)\). Since constraint (12) holds as an equality, then constraint (15) implies \( \rho(\hat{r}_e, \hat{C}_e) = 0 \). Suppose now \( \hat{C}_e = 0 \). In this case \( \hat{r}_e = \hat{r}_a^* \). Given Proposition 1, however, \( \hat{r}_a^* > \hat{r}_a^* \) which implies that constraint (13a) is violated. It follows that \( \hat{C}_e > 0 \).

(ii) Suppose now \( \hat{C}_e = \hat{C}_e \) and \( \hat{r}_e = \hat{r}_e \) for \( \mu = \frac{\hat{C}_e}{\mu} = \mu \). Notice that from the definition of \((\hat{r}_e, \hat{C}_e)\) and constraint (12a) it follows that \( \rho(\hat{r}_e, \hat{C}_e) = 0 \). Suppose now that there exists a pair of contracts \((\hat{r}_a^*, 0), (\hat{r}_a^*, \hat{C}_a^*)\) that solves Problem 1 for \( \mu = \frac{\hat{C}_e}{\mu} = \mu \) such that \( \rho(\hat{r}_a^*, \hat{C}_a^*) > 0 \). This contract must be such that \( \pi(\hat{r}_a^*, \hat{C}_a^*) \geq \pi(\hat{r}_e, \hat{C}_e) \).

Since \( \rho(\hat{r}_a^*, \hat{C}_a^*) = \rho(\hat{r}_a^*, \hat{C}_a^*) + \mu \rho(\hat{r}_a^*, \hat{C}_a^*) = 0 \), and \( \mu < \hat{\mu} \), we must have \( \rho(\hat{r}_a^*, 0) + \hat{\mu} \rho(\hat{r}_a^*, \hat{C}_a^*) = 0 \). Given the continuity of the \( \rho(\cdot) \) and \( \pi(\cdot) \) functions, then there exists a contract \((\hat{r}_a^*, \hat{C}_a^*) \) that solves Problem 1 for \( \mu = \frac{\hat{C}_e}{\mu} = \mu \) such that \( \pi(\hat{r}_a^*, \hat{C}_a^*) > \pi(\hat{r}_e, \hat{C}_e) \) which implies a contradiction. Let \( \hat{\mu} \) be the upper bound for \( \hat{C}_e = \hat{C}_e \) and \( \hat{r}_e = \hat{r}_e \). From the continuity of the \( \pi(\cdot) \) function it follows that \((\hat{r}_e, \hat{C}_e), (\hat{r}_a^*, \hat{C}_a^*) \) solves Problem 3 for \( \mu = \frac{\hat{C}_e}{\mu} = \hat{\mu} \).
(iii) Consider now constraints (13a) and (15). Together they implicitly define a function \( C_\ell(r_\delta) \) such that

\[
\frac{dC_\ell}{dr_\delta} = \left(1 - F(\tilde{R}_\delta, \delta) \right) \left[ \mu_\delta(1 - F(\tilde{R}_\delta, \delta)) + \mu_\ell(1 - F(\tilde{R}_\delta, \delta)) \right] > 0. \quad (A.1)
\]

Given constraint (12a) and the fact that \( d\pi_\delta/dr_\delta < 0 \), we have that the upper bound on \( r_\delta \) is given by \( r_\delta = r_\delta^* \). Given (A.1) it follows that \( \tilde{C}_\ell = C_\ell(r_\delta^*) \) is the upper bound on \( C_\ell \). Q.E.D.

**Proof of Proposition 6.** We consider now the case in which constraint (12) holds as a strict inequality. Since

\[
\pi_\ell(r, C) + \rho_\ell(r, C) = \int_0^T R \, dF(R, \theta) - \delta C - IB,
\]

we can rewrite constraint (13a) as \( \rho_\ell(r_\ell, 0) = \rho_\ell(r_\ell, C_\ell) + \delta C_\ell \). Substituting constraint (13a) into (15), Problem 3 can be rewritten as the problem of choosing the pair \((r_\ell, C_\ell)\) that maximizes

\[
\pi_\ell(r_\ell, C_\ell)
\]

subject to

\[
\mu_\delta \rho_\ell(r_\ell, C_\ell) + \mu_\delta \delta C_\ell + \mu_\ell \rho_\ell(r_\ell, C_\ell) = 0. \quad (A.3)
\]

Notice that, given eq. (5), constraint (A.3) implicitly defines a function \( r_\ell(C_\ell) \). Differentiating totally eq. (A.3) we obtain

\[
\frac{d r_\ell}{dC_\ell} = -\frac{\mu_\delta F(\tilde{R}_\ell, \theta_\delta) + \mu_\delta \delta + \mu_\ell F(\tilde{R}_\ell, \theta_\ell)}{\mu_\delta(1 - F(\tilde{R}_\ell, \theta_\delta)) + \mu_\ell(1 - F(\tilde{R}_\ell, \theta_\ell))}. \quad (A.4)
\]

Substituting \( r_\ell(C_\ell) \) into (A.2) and recalling (4), Problem (A.2)–(A.3) can be rewritten as the problem of choosing the level of collateral \( C_\ell \) that maximizes

\[
\int_{R_\ell(C_\ell)}^T (R - r_\ell(C_\ell)) B \, dF(R, \theta) - C_\ell F(\tilde{R}_\ell(C_\ell), \delta) - \delta C_\ell. \quad (A.5)
\]

The first-order conditions are

\[
-(\partial r_\ell/\partial C_\ell)(1 - F(\tilde{R}_\ell(C_\ell), \delta)) - F(\tilde{R}_\ell(C_\ell), \delta) - \delta < 0
\]

implies \( C_\ell = 0 \),

\[
-(\partial r_\ell/\partial C_\ell)(1 - F(\tilde{R}_\ell(C_\ell), \delta)) - F(\tilde{R}_\ell(C_\ell), \delta) - \delta = 0
\]

if \( C_\ell > 0 \).

Substituting (A.4) into (A.6) we have
\[-\mu_\delta(1+\delta)(1-F(\bar{R}(\bar{C}_\delta), \theta_\delta)) + [(1+\delta)\mu_\delta - \delta \mu_\tau]\]
\[\times (1-F(\bar{R}(C_\delta), \theta_\delta)) < 0. \tag{A.7}\]
implies \(C_\delta = 0\).

Notice that, given (2), if \(\mu_\delta > (1+\delta)\mu_\delta/\delta\), then both terms of eq. (A.7) are less than zero, which implies \(C_\delta = 0\). Q.E.D.

Proof of Proposition 7. Given that \(f(R, \theta_\delta)/f(R, \theta_\delta) > 1 - \delta \mu_\delta/(1+\delta)\mu_\delta\), then the second derivative of (A.5) is
\[
\{\mu_\delta(1+\delta)f(\bar{R}_\delta(C_\delta), \theta_\delta) - [(1+\delta)\mu_\delta - \delta \mu_\tau]f(\bar{R}_\delta(C_\delta), \theta_\delta)\}
\[\times [(b \partial r_\delta/\partial C_\delta - 1)] < 0, \tag{A.8}\]
so that (A.5) is concave and \(\bar{C}_\delta\) is an interior global maximum. Q.E.D.

Proof of Proposition 8. If (16) is satisfied, then also eq. (A.8) is satisfied. Therefore, differentiating eq. (A.6) totally, we obtain
\[
dC_\delta/d(\mu_\delta/\mu_\tau)
= \frac{\delta(1-F(\bar{R}_\delta, \theta_\delta))}{\{\mu_\delta(1+\delta)f(\bar{R}_\delta, \theta_\delta) - [(1+\delta)\mu_\delta - \delta \mu_\tau]f(\bar{R}_\delta, \theta_\delta)\}(\partial r_\delta B/\partial C_\delta - 1)} < 0.
\]
Q.E.D.

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